JUNIOR HIGH SCHOOL MATHEMATICS

VOSBURGH AND GENTLEMAN

FIRST COURSE

N. Young

Miss N. Young

JUNIOR HIGH SCHOOL MATHEMATICS

FIRST COURSE

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FIRST COURSE

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New York
THE MACMILLAN COMPANY
1922

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Set up and electrotyped. Published September, 1917.

Norwood Press
J. S. Cushing Co. — Berwick & Smith Co.
Norwood, Mass., U.S.A.

PREFACE

In planning a course in mathematics for the Junior High School, the authors have kept the following points in mind:

1. That instruction in mathematics in the seventh school year must of necessity begin at the point which the pupil may be assumed to have reached as a result of the training in arithmetic of the first six grades.

2. That the pupil has acquired, during this school period (Grades I–VI), an automatic mastery of certain number facts, a knowledge of the four processes in the field of integers and fractions, and an acquaintance with the facts and relations of the commonly used denominate units.

3. That the course in mathematics in the Junior High School should be of such content that it will bring the pupil in contact with adult activities which lend themselves to mathematical interpretation and afford him an opportunity for the exercise of his mathematical powers.

4. That the course in mathematics should be so administered that the pupil becomes habituated to the standards of the business world; *i.e.* (*a*) the computer must assume the responsibility for the correctness of his computation, (*b*) the computer must always, by check or by estimate, or by both, satisfy himself of the correctness of his work before it leaves his hands.

The attention of teachers is directed to the following features of the book:

1. The nature of the examples in addition, which are especially designed to remedy the pupil's weakness in handling the difficult combinations.

2. The simple forms of checking the work in addition and subtraction.

3. The emphasis placed on the estimate of the result and on rational methods of locating the decimal point in multiplication and division.

4. The early use of the equation as a simple mathematical tool.

5. The natural introduction to the idea of ratio, and the simple development of its use in the solution of analytic problems.

6. The rational development of measurement, and the extension of the decimal idea to this field, by the use of the protractor and decimalized ruler.

7. The interpretation of number data by means of graphs.

8. The revised treatment of the fundamentals of percentage, wherein the idea of *per cent* is first made clear.

9. The restriction of the treatment of percentage to simple direct applications.

10. The application of the equation to formulas of mensuration.

11. The emphasis on reasoning throughout the work in mensuration.

12. The selection of problem material from situations of interest to the child.

13. The presentation of all problem material in the form of direct statements followed by direct questions as to the results required.

WILLIAM LEDLEY VOSBURGH.
FREDERICK WILLIAM GENTLEMAN.

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JUNIOR HIGH SCHOOL MATHEMATICS

FIRST COURSE

CHAPTER I

REVIEWS OF ARITHMETIC

I. The Reading and Writing of Integers

1. In our system of notation, which is called the ***Hindu-Arabic system***, there are ten separate symbols used; namely, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. These symbols are called ***digits***, or ***figures***, or ***numerals***.

Each symbol by itself represents a single numerical value. Thus 2 = | | (two); 5 = | | | | | (five); etc.

2. When two or more of these symbols are written together, however, the number value then represented by each symbol depends also upon the place it occupies. Thus, using 2 and 5, the numbers twenty-five (25) or fifty-two (52) may be written. In the first number 2, by the place it occupies, represents two *tens*, 5 represents five *units;* in the second number 5 represents five *tens*, 2 represents two *units*. Using 2, 3, and 5 together six numbers may be written; as, 235, 253, 523, 532, 325, 352.

Noting the place that each figure occupies in each of the above numbers, tells the number that it represents.

3. The number value that a figure may express by reason of the place (or *order*) that it may occupy in a number is spoken of as its ***place value;*** the value that it denotes individually as its ***figure value.***

4. Since the symbol 0 (zero) when used alone does not express any number value it is called ***zero, cipher,*** or ***naught.*** When 0 (zero) is used with other symbols it may help to *place* them and so express number value; as, 230, 320, 302, 203, 023, 032.

Read each of the above numbers.

In which of them does the 0 help to express number values? Since the 0 serves no purpose in writing the last two numbers it is generally not written.

Read the following numbers:

560, 506, 605, 650, 2650, 2600, 2065, 2605, 2005.

Do not use *and* in reading whole numbers (integers).

5. For convenience in reading numbers of four or more figures, in the Hindu-Arabic system, the figures are generally separated by commas into groups of threes, called ***periods***.

The table at the top of page 3 shows the arrangement of *periods* and *orders* in our system (the Hindu-Arabic system) of writing numbers.

Read each of the numbers from the table.

How many orders in each period?

Name the orders in each period.

Name the periods, beginning at the right.

Read each number from the table.

Note that in reading a number each period is named after the number in that period has been read *except in units' period.*

BILLIONS (Orders)			MILLIONS (Orders)			THOUSANDS (Orders)			UNITS (Orders)		
hundreds	tens	units	hundreds	tens	units	hundreds	tens	units	hundreds	tens	units
							5	6	4	3	7
							5	9	2	0	0
						2	6	7	0	3	0
					7	0	8	1	0	0	2
					3	0	7	0	5	0	6
				2	5	4	3	0	0	0	4
				8	0	0	2	0	0	6	0
			2	5	0	3	0	2	0	5	6
			1	0	2	3	0	0	2	0	0
		2	5	6	0	4	0	0	0	3	0
	3	1	2	1	1	3	0	0	4	0	2
1	2	3	7	0	6	0	0	5	0	2	9
8	7	6	9	8	0	0	0	0	2	0	7

6. Summary: (1) There are ten separate symbols used in our system (the *Hindu-Arabic system*) of writing numbers. These symbols are called figures, numerals, or digits.

(2) Each figure has a *figure* value and a *place* value.

Thus, the figure value of zero is naught; but a zero that occupies a place in a number may be very important.

(3) The place each figure occupies in a number is called an *order*.

(4) In large numbers the figures are grouped by threes; these groups of threes are called *periods*.

(5) The names of the orders (single places) in each period are limited to three. Beginning at the right they are: Units (or ones), tens, and hundreds. (See the preceding table.)

(6) The names of the periods are units (or ones), thousands, millions, billions, trillions, etc. (See the table, p. 3.)

A million is a thousand thousand. A billion is a thousand million. A trillion is a thousand billion.

(7) The relation between successive orders is one of tens; that is, the number value of a figure is multiplied by ten when it is moved from one order to the next order to its left, and divided by ten when it is moved to the next order to its right.

(8) The two essential things are:

(*a*) To be able to read and write any number of three figures.

(*b*) To know the names of the periods.

Practice in these essentials is afforded by the exercises that follow.

EXERCISES

A. *Write in figures each of the following numbers:*

1. Two thousand, twenty-six.
2. Five thousand, two hundred one.
3. Thirty thousand, three hundred three.
4. Fifty-six thousand, nine.
5. Three hundred thousand, seventy-eight.
6. Nine hundred twenty-five thousand, nine.
7. Seventy thousand, seven.
8. Sixty thousand, sixty-three.
9. One million, two thousand, three.
10. Three million, four hundred three thousand, three hundred four.

B. *Write in words each of the following numbers:*

1. 526, 520, 503.
2. 3,063, 4,206, 6,001.
3. 1,607, 6,050.
4. 8,084, 7,072.
5. 23,006.
6. 50,705.
7. 203,400.
8. 902,001.
9. 927,006.
10. 3,020,052.
11. 20,050,500.
12. 2,201,002.
13. 26,000,038.
14. 6,067,308.
15. 60,006,006.
16. 320,000,200.
17. 256,676,000.
18. 3,000,780,500.

C. *Express in words and also in figures each of the following numbers:*

1. The population of your state.
2. The population of the United States.
3. The number of feet in one mile; also the number of inches in one mile.
4. The area of your state.
5. The area of New England; the Middle States; the South Atlantic States; the North Central States; the South Central States; the Western States.

II. The Reading and Writing of Decimals

7. A ***decimal fraction***, or a ***decimal***, is a fraction whose denominator is 10, 100, or 1000, etc. The denominator is not written, but is expressed by the decimal point (.) placed in front of the numerator.

Thus:

.3 expresses $\frac{3}{10}$.

.23 expresses $\frac{23}{100}$.

.234 expresses $\frac{234}{1000}$.

8. To express some decimals one or more zeros have to be inserted between the decimal point and the numerator.

Thus:

$\frac{3}{100}$ is written .03

$\frac{3}{1000}$ is written .003

$\frac{23}{1000}$ is written .023

9. An integer and a decimal together, called a ***mixed decimal***, express a ***mixed number***.

Thus:

3.2 expresses $3\frac{2}{10}$.

10. *To read a decimal*, read it as a whole number and then give it the name of the last decimal place.

Thus:

.206 is read two hundred six *thousandths*.

.0322 is read three hundred twenty-two *ten-thousandths*.

To read a mixed decimal, read the integral part (the whole number) as usual, use *and* at the decimal point, and then read the decimal.

Thus:

7.206 is read seven *and* two hundred six *thousandths*.

212.2581 is read two hundred twelve *and* two thousand five hundred eighty-one *ten-thousandths*.

11. The table below shows the arrangement of *orders* in our decimal system.

Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths	Thousandths	Ten-thousandths
						.	6			
						.	2	6		
						.	2	2	6	
						.	2	6	0	
						.	0	3	5	8
					2	.	4	6		
					3	.	0	0	2	
				1	8	.	0	5	9	
			2	5	0	.	0	2	5	
			3	0	7	.	2	1	6	
			5	0	0	.	2	3	7	6
		2	5	2	6	.	2	3	7	6
		5	0	0	2	.	3	0	0	4
2	5	0	0	0	0	.	0	3	7	8
	5	6	0	0	0	.	0	3	1	2

Beginning at the decimal point, name the decimal orders. Read each number given in the table.

EXERCISES

Read each of the following numbers:

1. .2, .02, .023, .020, .0026

2. 3.2, 14.02, 5.026, 5.202, 6.305

3. 256.203, 102.2603, 200.031, 2002.02

4. 3010.010, 5002.002, 30.030, 43.043

5. 4006.300, 4050.003, 32.0040, 70.0070

12. *To write a decimal in figures:*

(1) Write the numerator as you write any whole number.

(2) Place the units' figure of this whole number in the decimal order given.

(3) Insert zeros, if needed, between the decimal point and numerator.

Thus, to write in figures, twenty-six *thousandths*,

(1) Write 26,

(2) Place 6 in *thousandth's* order,

(3) Insert one zero.

It then appears as .026

EXERCISES

Write in figures the following:

1. Twenty-five *hundredths*.
2. Three hundred two *thousandths*.
3. Two *thousandths*.
4. Two and two *thousandths*.
5. Two hundred five and five *hundredths*.
6. Twenty-two *thousandths*.
7. Sixty-two and twenty-three *ten-thousandths*.
8. Two hundred and twenty *hundredths*.
9. Two hundred and two *hundred-thousandths*.
10. Two hundred two *hundred-thousandths*.

13. In business practice and scientific work the pure decimal, .24, is commonly written 0.24; that is, the absence of any integral part is emphasized by writing 0 in units' place.

Read each of the following quantities:

\$0.56; 0.2 in.; 0.72 ft.; 0.125 in.; 0.80 ft.

III. The Roman System

14. In the ***Roman system***, seven separate symbols (capital letters) are used, namely:

I = one	C = one hundred
V = five	D = five hundred
X = ten	M = one thousand
L = fifty	

15. These letters are combined as follows:

(1) If a letter is repeated, its value is repeated; for example,

II = 2, XX = 20, XXIII = 23, CCXXII = 222.

(2) If a letter is placed before one of greater value, its value is taken from that of the greater; for example,

IV = 4, XC = 90, XIX = 19, XL = 40.

(3) If a letter is placed after one of greater value, its value is added to that of the greater; for example,

VII = 7, CXV = 115.

(4) If a bar is placed over a letter, its value is increased a thousand fold; for example,

$\overline{\text{V}} = 5000.$

The Roman system is used to-day only in such exceptional cases as chapter headings, dates on monuments, etc.

EXERCISES

1. *Read the following numbers:*

XC, XXIV, LII, XLII, XLIV, XCIV, CCLII, MCII, CXXXVI, MD, MDCXX, MDCCLXXVI, MCMXVI.

2. *Write the following numbers in the Roman system:* 36, 47, 51, 90, 142, 196, 1660, 1781, 1812, 1900, 1917.

IV. The Four Fundamental Processes with Integers and Decimals

16. To meet the demands of the business world to-day the pupil must form the habit of checking his work.

To check his work he must do two things:

1. Leave a record of the successive steps of the computation.

2. Always go over the computation a second time in some different manner, or order, if possible.

17. Addition. The ***addends*** are the numbers added. The ***sum*** or ***amount*** is the result obtained.

In writing down numbers to be added it is necessary to make the figures legible and to write figures of the same order in the same vertical columns. This is especially important in work that is to be read by others.

As addition is the process most frequently used, the habit of checking is most important here.

A simple method of recording the successive steps (column sums) in addition is shown below.

Also a second addition of each column (downward), and the checking up of the column sum by the check mark (√) is shown.

Example 1.

8,786	(Record of work)
5,674	3 3√
8,789	4 0√
6,587	3 8√
8,967	3 8√
38,803	

EXPLANATION. The sum of units column is 33; 3 is carried to the next column and the sum 40 is set down under the 33; 4 is carried to the next column and the sum 38 is set down under the 40; 3 is carried to the next column and the sum 38 is set down under the 38. In the "Record of work" the sum of two columns is 403, of three 3803, of all 38,803.

CHECK. If the columns are added upward, check by adding downward. After each column has been added downward and found to be correct, put a check mark (√) by the sum for that column.

This method of recording the successive sums of columns makes it possible, in checking another person's work, to compare the answers obtained at the end of the corresponding columns as the work progresses.

EXAMPLE 2.

$5,462.28	
379.87	5 4√
4,687.66	4 7√
278.89	5 7√
86.38	6 3√
5,787.76	3 9√
2,576.67	2 4√
25,678.23	4√
$44,937.74	

NOTE. The decimal point is not regarded in the check record.

While many business offices now use adding machines to add such columns, a great many sums must be done without the machines. In any case, the work of machines must be checked frequently.

EXERCISES

Find the sums of each of the following examples. Leave a record of the column sums as shown in the preceding examples. Check each answer by the method shown above.

1.	2.	3.	4.
695,498	987,456	659,789	6,857
765,489	576,897	675	479,584
987,654	768,498	7,598	687,489
456,789	895,765	76,958	95,795
856,497	684,976	957,869	48
774,596	567,895	697	87,654
698,975	456,789	89	558,789
	987,654	69,858	5,371
		6,989	678,995
		598,567	785,968

5.	6.	7.	8.
72,308	84,165	612,384	4,615
6,213	125,004	4,126	27
73	269	27,000	67,903
1,208	1,300	5,269	729
5,799	875,695	879,548	894
89,746	5,678	897	6,945
559,876	756	684,578	587
788	69,587	485	876,694
6,546	6,958	87,699	945,889
645,678	87,569	694,877	95,674
57,788	569	9,789	8,765
578,657	695,875	65,876	569
4,578	58,569	8,965	6,898
455,789	989,755	878	76,548

9.	10.	11.	12.
874,695	856,479	$9465.78	$94.85
47,768	648,975	69.45	95.76
798,549	597,869	964.85	467.97
6,794	964	96.84	7948.69
957,678	9,765	9475.86	8589.44
764,955	658	6998.45	4756.58
9,586	8,954	874.45	6.97
7,868	965,879	69.47	68.45
645,759	879,648	896.74	474.98
788,484	495,765	8467.95	8959.76

13.	14.	15.	16.
$6957.58	$8756.49	$5768.97	$8868.58
9476.85	9874.68	4985.64	5678.97
574.96	5547.86	659.49	8497.66
7546.89	4756.58	468.98	9954.78
68.75	8978.59	7654.89	7877.79
5789.46	4695.86	6789.54	6559.98
645.84	7884.47	98.57	8868.65
9876.54	7649.55	76.76	6766.79
4567.89	9477.68	957.68	8747.97
8659.74	8968.86	5784.99	9856.89

17. Verify the totals in the following table.

WORLD'S PRODUCTION OF COTTON

Countries	1914–1915	1913–1914	1912–1913	1911–1912	1910–1911
	Bales	Bales	Bales	Bales	Bales
United States	14,766,467	14,494,762	13,943,220	15,683,945	11,804,749
East Indies	3,337,000	4,592,149	3,468,407	3,107,660	3,235,748
Egypt	1,200,000	1,439,802	1,416,352	1,396,474	1,415,711
Brazil, &c.	240,000	387,947	370,000	341,836	406,529
Total	19,543,467	20,914,660	19,197,979	20,529,915	16,862,737

18. The increase in railway track mileage in the United States for the years 1900 and 1910 was as follows:

	1900	1910	(Total for Each District)
Eastern District	64,537 (miles)	75,129 (miles)	
Southern District	33,117 (miles)	43,694 (miles)	
Western District	108,977 (miles)	147,362 (miles)	
Total for each year . . .			

(*a*) Find the total for 1900; 1910.

(*b*) Find the total for each district.

(*c*) Check by comparing the sums of the totals.

19. The increase in acreage of improved farm land in the United States for the years 1900 and 1910 was as follows:

	1900	1910	(Total for Each District)
Eastern District	90,921,000 (acres)	89,641,000 (acres)	
Southern District	82,061,000 (acres)	88,353,000 (acres)	
Western District	241,516,000 (acres)	300,458,000 (acres)	
Total for each year . . .			

(*a*) Find the total for 1900; 1910.

(*b*) Find the total for each district.

(*c*) Check by comparing the sums of the totals.

18. Subtraction. The ***subtrahend*** is the number taken away or subtracted.

The ***minuend*** is the number from which you take the subtrahend.

The result of the subtraction is called the ***remainder*** or ***difference***.

The "addition" or "shop" method of subtraction is shown below. This method has the following advantages over other methods:

1. It is a self-checking method since that number is found which if added to the subtrahend will give the minuend.

2. There is no "borrowing" difficulty.

3. It is the universal method of "making change" in the business world.

EXAMPLE 1.

	√√√√√	EXPLANATION OF WORK.
WORK.	56,134	6 and ? are 14, write **8**, carry 1 to 5
	23,456	6 and ? are 13, write **7**, carry 1 to 4
	32,678	5 and ? are 11, write **6**, carry 1 to 3
		4 and ? are 6, write **2**, (no carrying)
		2 and ? are 5, write **3**.

EXPLANATION OF CHECK.

Add upward the answer to the subtrahend, thus:

8 + 6 = 14, put check (√) over 4; carry 1 to 7
8 + 5 = 13, put check (√) over 3; carry 1 to 6
7 + 4 = 11, put check (√) over 1; carry 1 to 2
3 + 3 = 6, put check (√) over 6; (nothing to carry)
3 + 2 = 5, put check (√) over 5.

EXAMPLE 2.

	√√√√	EXPLANATION OF WORK.
WORK.	5002	5 and ? are 12, write **7**, carry 1 to 3
	2735	4 and ? are 10, write **6**, carry 1 to 7
	2267	8 and ? are 10, write **2**, carry 1 to 3
		3 and ? are 5, write **2**.

EXPLANATION OF CHECK.

Add upward the answer to the subtrahend, thus:

7 + 5 = 12, put check (√) over 2; carry 1 to 6
7 + 3 = 10, put check (√) over 0; carry 1 to 2
3 + 7 = 10, put check (√) over 0; carry 1 to 2
3 + 2 = 5, put check (√) over 5.

This method of checking is independent of the method of subtraction used. Identical errors are practically impossible.

EXERCISES

Find the difference or remainder in each of the following exercises. Check each answer by adding it to the subtrahend to see if this sum gives the minuend.

1.	**3.**	**5.**	**7.**	**9.**
207,568	$3,086,120	$250,000	75,129	108,977
137,243	1,536,376	187,441	43,694	33,117
2.	**4.**	**6.**	**8.**	**10.**
124,723	232,743	$350.	147,362	210,001
66,069	21,079	187.31	75,129	64,577

From the first amount take the second in each of the following exercises:

11. $201.10, $87.67
12. $700.03, $157.48
13. $3100.02, $837.16
14. $82,102.00, $71,191.17

Take the first number from the second in each of the following exercises:

15. 272.3, 1201.1
16. 394.02, 1002.2
17. 738.16, 2100.6
18. 56.25, 3103.52
19. 867.72, 2130
20. 1027.2, 1410.96

Find the difference between each of the following pairs of numbers:

21. 66.3 and 43.7
22. 160.5 and 124.87
23. 1523.4 and 75.88
24. 2001 and 782.6
25. 32.563 and 325.63
26. 0.0246 and 0.0057

Find the excess of the first number over the second in each of the following exercises:

27. 1215.76 over 87.876
28. 2103.17 over 897.29
29. $1567.13 over $999.87
30. $2,112,000 over $787,026
31. 3,010,001 over 1,357,973
32. 0.0206 over 0.00678

19. Multiplication. The ***multiplicand*** is the number to be multiplied.

The ***multiplier*** is the number by which you multiply.

The result of the multiplication is called the ***product***.

In multiplication, especially of decimals, you should make a round number *estimate* of your product *before* performing the work. You should write down this estimate when you make it and submit it as a part of the record of your work.

You should use this estimate to check up the location of the decimal point in your product, thus avoiding serious, as well as absurd, blunders.

For an accurate check in multiplication, you should do the work a second time, using the old multiplicand as the new multiplier and the old multiplier as the new multiplicand.

PROBLEM. — *Find in pounds the weight of* 82.6 *cu. ft. of water.* [NOTE. — 1 cu. ft. of water weighs 62.5 lb.]

METHOD (*a*)		METHOD (*b*)
62.5		62.5
82.6	ESTIMATE	82.6
3750		5000.
1250	$85 \times 60 = 5100$	125.0
5000		37.50
5162.50		5162.50

EXPLANATION OF METHOD (*a*). This method is older and more widely used than method (*b*). The partial products are formed by multiplying the multiplicand by each digit in the multiplier, in order, proceeding from *right* to *left*. These partial products are set down under each other, each being advanced one place to the left; and they are then added to form the final product.

In the final product we locate the decimal point by the following rule: The number of digits to the right of the decimal point is equal to the total number of digits to the right of the decimal points in both multiplicand and multiplier.

EXPLANATION OF METHOD (*b*). In multiplying by method (*b*) we use the figures of the multiplier in order from *left* to *right* instead of from right to left as usual.

To obtain the first partial product the figure 8 of the multiplier is used first. Now 8 here represents 8 *tens*, that is, 80; so we place the decimal point in this first partial product after the 5000 (estimate was 5100).

To obtain the next partial product we multiply by 2, placing the figure first obtained in this partial product one column to the *right* of the preceding product.

To obtain the last partial product we multiply by 6, placing the figure first obtained in this partial product again one column to the *right* of the preceding product.

In this method we locate the decimal point by estimate and common sense early in the work, that is, in the first partial product. Carelessness and errors in its location in the final product are thereby prevented.

ACCURATE CHECK.

METHOD (*a*)		METHOD (*b*)
82.6	ESTIMATE	82.6
62.5		62.5
4130	$60 \times 85 = 5100.$	4956.
1652	or	165.2
4956	$65 \times 80 = 5200.$	41.30
5162.50		5162.50

EXERCISES

Carry out each of the following indicated multiplications:

1. 32×3.142
2. 53×39.37
3. 64×62.48
4. 96×3.142
5. 87×3.375
6. 45×8.63
7. 78×48.83
8. 29×39.83
9. 68×62.479
10. 83×2150.42
11. 7.6×285
12. 5.3×483
13. $.37 \times 327$
14. $.59 \times 964$
15. 2.63×7358
16. 3.7×56.78
17. 5.2×159.24
18. 6.23×354.85
19. $.587 \times 385.4$
20. $.38 \times 56.37$

20. Division. The ***dividend*** is the number to be measured or divided.

The ***divisor*** is the number by which the dividend is measured or divided.

The result of the division is the ***quotient***.

EXAMPLE 1. *Divide* 37,647 *by* 89.

WORK.

$$\begin{array}{r} 423 \\ 89\overline{)37647} \\ \underline{356} \\ 204 \\ \underline{178} \\ 267 \\ \underline{267} \end{array}$$

CHECK.

$$\begin{array}{r} 423 \\ \underline{89} \\ 3807 \\ \underline{3384} \\ 37647 \end{array}$$

It is clear that division is really the reverse of multiplication; the process is also the reverse of the process for multiplication.

For an accurate check in division, multiply the quotient by the divisor; add the remainder, if any, to this product. The result should be the dividend.

EXAMPLE 2. *Divide* 451.36 *by* 6.2.

METHOD (*a*)

For this example multiply both numbers by 10 to make the divisor a whole number; then divide as in integers. $451.36 \div 6.2$ becomes $4513.6 \div 62$.

Place the decimal point in the quotient over the decimal point in the dividend.

METHOD (*b*)

Estimate the size of the answer before dividing, as $450 \div 6 =$ about 70.

Place the decimal point in the quotient by comparison with the estimate.

WORK. 72.8	WORK. 72.8
62)4513.6	6.2)451.36
434	434
173	173
124	124
496	496
496	496
CHECK. 72.8	CHECK. 72.8
6.2	6.2
1456	436.8
4368	14.56
451.36	451.36

EXERCISES

Carry out each of the following indicated divisions to *four* figures:

1. $82\overline{)166{,}330}$
2. $71\overline{)568{,}279}$
3. $91\overline{)822{,}134}$
4. $89\overline{)2911.32}$
5. $98\overline{)88{,}437}$
6. $89\overline{)359.09}$
7. $66\overline{)71.982}$
8. $9.9\overline{)812.655}$
9. $5.3\overline{)162.579}$
10. $8.7\overline{)43{,}365.3}$
11. $322\overline{)877{,}846}$
12. $224\overline{)498{,}263}$
13. $42.1\overline{)97{,}978.3}$
14. $2.13\overline{)489.90}$
15. $4.21\overline{)127.42}$
16. $11.4\overline{)47.138}$
17. $4.71\overline{)12.742}$
18. $59.2\overline{)784.436}$
19. $8.62\overline{)1943.936}$
20. $7.61\overline{)1846.363}$
21. $6.32\overline{)182.03}$
22. $741\overline{)21{,}134}$
23. $852\overline{)72{,}260}$
24. $354\overline{)769{,}234}$
25. $236\overline{)702{,}263}$
26. $3.21\overline{)7022.63}$

V. Common Fractions

21. Addition of Fractions.

Example 1. *Find the sum of* $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, *and* $\frac{11}{16}$.

$$\begin{aligned} \tfrac{3}{4} &= \tfrac{36}{48} \\ \tfrac{5}{6} &= \tfrac{40}{48} \\ \tfrac{7}{8} &= \tfrac{42}{48} \\ \tfrac{11}{16} &= \tfrac{33}{48} \\ \hline & \tfrac{151}{48} = 3\tfrac{7}{48} \end{aligned}$$

Example 2. *Find the sum of* $17\frac{3}{8}$, $16\frac{5}{16}$, *and* $31\frac{27}{32}$.

$$\begin{aligned} 17\tfrac{3}{8} &= 17\tfrac{12}{32} \\ 16\tfrac{5}{16} &= 16\tfrac{10}{32} \\ 31\tfrac{27}{32} &= 31\tfrac{27}{32} \\ \hline & 64\tfrac{49}{32} = 65\tfrac{17}{32} \end{aligned}$$

To find the *lowest common denominator* of several fractions in the most economical way:

(*a*) Take the largest denominator (in Example 1: 16).

(*b*) Try multiples of it (in Example 1: 16, 32, 48, etc.).

(*c*) Choose the smallest multiple that will contain each of the other denominators (in Example $1 : 3 \times 16$ or 48).

EXERCISES

Carry out each of the following indicated additions:

1. $\frac{3}{4} + \frac{5}{6} + \frac{7}{8}$.
2. $\frac{3}{4} + \frac{5}{9} + \frac{5}{6}$.
3. $\frac{5}{6} + \frac{7}{10} + \frac{3}{4}$.
4. $\frac{3}{4} + \frac{5}{6} + \frac{9}{16}$.
5. $\frac{7}{8} + \frac{1}{4} + \frac{9}{10}$.
6. $\frac{5}{12} + \frac{5}{8} + \frac{3}{4}$.
7. $\frac{3}{4} + \frac{5}{8} + \frac{3}{14}$.
8. $\frac{3}{10} + \frac{5}{12} + \frac{1}{4}$.
9. $\frac{9}{10} + \frac{1}{4} + \frac{15}{16}$.
10. $\frac{11}{12} + \frac{3}{4} + \frac{13}{16}$.
11. $\frac{5}{6} + \frac{3}{8} + \frac{7}{16}$.
12. $\frac{3}{8} + \frac{8}{9} + \frac{5}{12}$.
13. $\frac{7}{9} + \frac{3}{10} + \frac{2}{15}$.
14. $\frac{3}{4} + \frac{5}{6} + \frac{7}{8} + \frac{5}{16}$.
15. $\frac{5}{6} + \frac{3}{8} + \frac{3}{16} + \frac{5}{16}$.
16. $\frac{5}{9} + \frac{7}{10} + \frac{11}{12}$.

17. $\frac{7}{10} + \frac{3}{4} + \frac{11}{14}$.
18. $\frac{1}{2} + \frac{7}{8} + \frac{5}{16} + \frac{7}{32}$.
19. $\frac{1}{2} + \frac{7}{8} + \frac{15}{16} + \frac{17}{32}$.
20. $\frac{3}{4} + \frac{5}{8} + \frac{13}{16} + \frac{25}{32}$.
21. $26\frac{1}{3} + 52\frac{5}{6} + 12\frac{1}{4}$.
22. $37\frac{3}{4} + 16\frac{5}{8} + 25\frac{1}{6}$.
23. $54\frac{5}{9} + 8\frac{2}{3} + 16\frac{5}{6}$.
24. $23\frac{1}{4} + 56\frac{7}{8} + 18\frac{5}{16}$.

22. Subtraction of Fractions.

EXAMPLE 1. *Subtract* $9\frac{2}{3}$ *from* $15\frac{3}{4}$.

$$\begin{array}{r} 15\frac{3}{4} = 15\frac{9}{12} \\ 9\frac{2}{3} = 9\frac{8}{12} \\ \hline 6\frac{1}{12} \end{array}$$

EXAMPLE 2. *Subtract* $16\frac{3}{4}$ *from* $23\frac{1}{3}$.

$$\begin{array}{r} 23\frac{1}{3} = 23\frac{4}{12} = 22\frac{16}{12} \\ 16\frac{3}{4} = 16\frac{9}{12} = 16\frac{9}{12} \\ \hline 6\frac{7}{12} \end{array}$$

EXERCISES

Carry out each of the following indicated subtractions:

1. From $\frac{15}{16}$ take $\frac{5}{8}$.
2. From $\frac{5}{6}$ take $\frac{3}{4}$.
3. From $\frac{9}{16}$ take $\frac{5}{32}$.
4. From 2 take $\frac{7}{8}$.
5. From 3 take $\frac{3}{4}$.
6. From $5\frac{14}{16}$ take $\frac{5}{8}$.
7. From $7\frac{3}{4}$ take $3\frac{5}{8}$.
8. From $9\frac{5}{6}$ take $2\frac{5}{8}$.
9. From $11\frac{2}{3}$ take $6\frac{3}{4}$.
10. From $12\frac{1}{4}$ take $7\frac{5}{8}$.
11. From $27\frac{3}{8}$ take $17\frac{5}{6}$.
12. From $31\frac{1}{4}$ take $29\frac{7}{8}$.
13. From $45\frac{3}{16}$ take $26\frac{7}{8}$.
14. From $120\frac{5}{8}$ take $97\frac{5}{6}$.
15. From $211\frac{3}{8}$ take $57\frac{3}{10}$.
16. From $312\frac{3}{4}$ take $34\frac{3}{8}$.

23. Multiplication of Fractions.

EXAMPLE 1. *Multiply* $\frac{8}{9}$ *by* 1232.

$$1232 \times \tfrac{8}{9} = \tfrac{9856}{9} = 1095\tfrac{1}{9}$$

EXAMPLE 2. *Find* $\frac{7}{8}$ *of* 6928.

$$\frac{7}{\cancel{8}} \text{ of } \frac{866}{\cancel{6928}} = 6062$$

EXAMPLE 3. *Multiply* $56\frac{2}{3}$ *by* 8.

$$\begin{array}{r} 56\frac{2}{3} \\ \times\quad 8 \\ \hline 448 \\ \frac{5}{3} \\ \hline 453\frac{1}{3} \end{array}$$

EXAMPLE 4. *Multiply* $12\frac{2}{3}$ *by* $3\frac{1}{2}$.

METHOD (*a*)

$$\begin{array}{r} 12\frac{2}{3} \\ \times\quad 3\frac{1}{2} \\ \hline 36 \\ 2\ (3 \times \frac{2}{3}) \\ 6\ (\frac{1}{2} \text{ of } 12) \\ \frac{1}{3}\ (\frac{1}{2} \text{ of } \frac{2}{3}) \\ \hline 44\frac{1}{3} \end{array}$$

METHOD (*b*)

$$12\frac{2}{3} = \frac{38}{3}$$
$$3\frac{1}{2} = \frac{7}{2}$$
$$\frac{\overset{19}{\cancel{38}}}{\underset{3}{\cancel{3}}} \times \frac{7}{\underset{2}{\cancel{2}}} = \frac{133}{3}$$
$$= 44\frac{1}{3}$$

EXERCISES

A. Carry out each of the following multiplications:

1. $564 \times \frac{7}{8}$.
2. $348 \times \frac{5}{6}$.
3. $756 \times \frac{7}{12}$.
4. $313 \times \frac{3}{4}$.
5. $1232 \times \frac{8}{9}$.
6. $26\frac{3}{4} \times 9$.
7. $37\frac{2}{3} \times 8$.
8. $76\frac{3}{8} \times 12$.
9. $84\frac{7}{9} \times 15$.
10. $\frac{5}{6}$ of $\frac{7}{12}$.
11. $\frac{8}{9}$ of $\frac{11}{12}$.
12. $\frac{9}{10}$ of $\frac{15}{16}$.

B. In the following exercises, both multiplicand and multiplier are mixed numbers. Find the product in each case.

1. $8\frac{2}{3} \times 6\frac{3}{4}$.
2. $12\frac{3}{5} \times 20\frac{3}{8}$.
3. $24\frac{5}{6} \times 3\frac{3}{4}$.
4. $61\frac{1}{8} \times 9\frac{1}{2}$.
5. $29\frac{3}{8} \times 8\frac{1}{3}$.
6. $1564\frac{3}{4} \times 7\frac{1}{3}$.
7. $2875\frac{2}{3} \times 8\frac{2}{3}$.
8. $7568\frac{3}{4} \times 5\frac{1}{2}$.

24. Division of Fractions.

EXAMPLE 1. *Divide* $\frac{17}{18}$ *by* 3.

$$\frac{17}{18} \div 3 = \frac{17}{18} \times \frac{1}{3} = \frac{17}{54}$$

EXAMPLE 2. *Divide* $\frac{15}{16}$ *by* $\frac{3}{4}$.

$$\frac{15}{16} \div \frac{3}{4} = \frac{\overset{5}{\cancel{15}}}{\underset{4}{\cancel{16}}} \times \frac{\cancel{4}}{\cancel{3}} = \frac{5}{4} = 1\frac{1}{4}$$

EXAMPLE 3. *Divide* $5\frac{1}{3}$ *by* $3\frac{1}{2}$.

$$5\frac{1}{3} \div 3\frac{1}{2} = \frac{16}{3} \div \frac{7}{2} = \frac{16}{3} \times \frac{2}{7} = \frac{32}{21} = 1\frac{11}{21}$$

EXERCISES

Carry out each of the following indicated divisions:

1.	$\frac{8}{9} \div 2$.	**7.**	$8 \div \frac{3}{4}$.	**13.**	$\frac{3}{5} \div \frac{3}{7}$.	**19.**	$21\frac{5}{6} \div 6$.
2.	$\frac{3}{8} \div 2$.	**8.**	$9 \div \frac{2}{3}$.	**14.**	$\frac{17}{24} \div \frac{8}{9}$.	**20.**	$24\frac{7}{8} \div 8$.
3.	$\frac{7}{8} \div 2$.	**9.**	$12 \div \frac{5}{6}$.	**15.**	$\frac{19}{32} \div \frac{5}{8}$.	**21.**	$6\frac{2}{3} \div 2\frac{1}{2}$.
4.	$\frac{7}{8} \div 4$.	**10.**	$\frac{7}{8} \div \frac{1}{4}$.	**16.**	$5\frac{1}{3} \div 3$.	**22.**	$13\frac{1}{2} \div 3\frac{3}{4}$.
5.	$\frac{3}{4} \div 4$.	**11.**	$\frac{15}{16} \div \frac{3}{4}$.	**17.**	$16\frac{3}{4} \div 4$.	**23.**	$16\frac{7}{8} \div 2\frac{1}{2}$.
6.	$3 \div \frac{7}{8}$.	**12.**	$\frac{9}{12} \div \frac{3}{4}$.	**18.**	$18\frac{2}{3} \div 6$.	**24.**	$25\frac{4}{5} \div 3\frac{1}{3}$.

25. Sums of Common and Decimal Fractions. To find the sum of a common fraction and a decimal fraction, either reduce both of them to common fractions, or both to decimal fractions.

EXERCISES

Add together all of the numbers in each of the following exercises:

1. $12\frac{1}{2}$, 13.2, 17.8, $16\frac{1}{5}$, $8\frac{4}{5}$.

2. $85\frac{1}{3}$, $78\frac{1}{6}$, 38.25, 96.5, $87\frac{1}{3}$.

3. $69\frac{3}{8}$, 75.5, 64.5, $68\frac{3}{8}$, $56\frac{7}{8}$.

4. 29.75, $69\frac{7}{8}$, $79\frac{3}{8}$, $46\frac{1}{8}$, 28.25

5. $83\frac{5}{6}$, $47\frac{2}{3}$, 87.25, 67.125, $81\frac{1}{3}$.

6. 46.3, 78.27, $67\frac{4}{5}$, 78.2, $98\frac{3}{5}$.

7. $86\frac{2}{3}$, $58\frac{5}{6}$, 92.8, 87.9, $69\frac{1}{3}$.

8. $95\frac{4}{5}$, 67.8, 89.3, $76\frac{1}{4}$, $56\frac{3}{8}$.

9. $77\frac{7}{10}$, 66.6, $88\frac{4}{5}$, $55\frac{1}{2}$, 99.9

10. 316.3, $21\frac{2}{5}$, $817\frac{5}{6}$, $86\frac{3}{4}$, 718.7

MISCELLANEOUS PROBLEMS FOR CHAPTER I

1. In a class of 18 pupils the records on a certain test were: 97, 95, 93, 92, 86, 86, 85, 82, 78, 78, 74, 74, 72, 71, 67, 62, 57, 55. Find the total and the average.

[NOTE. To find the average divide the total by the number of pupils.]

2. Copy the following fifteen items: 235, 341, 21, 3224, 345, 67, 897, 78, 896, 475, 2376, 54, 96, 678, 87. Find the total and the average.

3. At \$42 a dozen what will $8\frac{3}{4}$ dozen hats cost?

4. The number of Irish-born residents in the United States in 1880 was 1,854,815, and in 1890, 1,871,509. What was the increase?

5. A pays \$7215 for a farm of 78 acres. What does he pay per acre?

6. A merchant paid \$8.75 per barrel for 562 barrels of flour. How much did the flour cost him?

7. Find the product of 19,279 multiplied by 26.52.

8. The cost of maintenance of the Public Library in Boston for 1914 was \$400,485; this was \$19,636 more than in 1913. What was the cost in 1913?

9. It takes $\frac{7}{8}$ yd. of braid for a coat; how many yards are needed for 12 coats? At 27 cents per yard what will the braid for the 12 coats cost?

10. A bolt of cloth containing 55 yards is cut into overcoats. Allowing $3\frac{1}{4}$ yards for a coat, how many coats can be cut? How many yards will be wasted?

11. A farm of 160 acres was sold for \$20,675, including stock valued at \$1500, and farming implements at \$775. Find the price per acre for the land itself.

12. How many times can a pail holding 3 pints be filled from a jar containing 3 gallons 3 quarts?

13. John paid \$2.00 for 8 dozen oranges. He sold these oranges at the Fair at the rate of two for five cents. How much did he make?

14. At 75 cents a square yard, find the cost of 62 square yards of linoleum.

15. A train averages 28 miles per hour. How far will it go in $12\frac{2}{5}$ hours?

16. A cubic foot of water weighs 62.5 pounds. What is the weight of 264 cu. ft. of water?

17. How many cu. ft. in 215,042 cu. in.?

18. An acre of land costs \$225. Find the cost of $\frac{5}{8}$ of an acre.

19. A lot of land containing 5250 sq. ft. is purchased at 18 cents a square foot. Find the cost.

20. When eggs are retailing at 54 cents per dozen, how much should you pay for 32 eggs?

21. When velvet is selling at \$4.50 a yard, how much should be paid for $12\frac{3}{8}$ yd.?

22. Find the cost of 225 tons of coal at \$7.90 per ton.

23. A farmer raised 1200 bu. of potatoes. He sold $\frac{3}{8}$ of them at \$1.50 per bushel; the remainder at \$1.25 per bushel. How much did he receive for his crop of potatoes?

24. A truck gardener raised 15,000 bunches of celery. He sold $\frac{3}{5}$ of them at 12 cents a bunch; the remainder at 10 cents a bunch. How much did he receive for his crop of celery?

25. In a certain school of 54 pupils $\frac{5}{6}$ are girls. How many boys are in this school?

26. A carload of 28 western horses cost on an average \$182.50 each. Find the cost of the carload.

27. If $\frac{5}{8}$ yd. of silk cost \$2.00, what is the price of a yard?

28. For $\frac{7}{8}$ yd. of velvet \$4.20 is paid. What price is this per yard?

29. For $1\frac{1}{4}$ yd. of velvet \$6.00 is paid. What price is this per yard?

30. Find the total number of yards in the following pieces of cloth: $25\frac{3}{4}$ yd., $78\frac{3}{4}$ yd., $56\frac{5}{8}$ yd., $46\frac{3}{8}$ yd., $65\frac{7}{8}$ yd., $38\frac{3}{8}$ yd.

31. At $\$24\frac{3}{4}$ each, find the cost of 17 tables.

32. At $\$37\frac{5}{8}$ each, find the cost of 24 tables.

33. A distance of 125 miles is covered by automobile in $4\frac{1}{2}$ hours. What is the average distance traveled per hour?

34. A turkey weighing $16\frac{3}{8}$ lb. is bought at 44 cents a pound. How much is paid for this turkey?

35. A man pays \$426 for the rent of an apartment for one year. What does he pay a month?

36. A man pays \$4 a week for the rent of a house. What does he pay a month? (52 weeks in a year.)

37. A roast of beef weighing $8\frac{3}{4}$ lb. is bought at 33¢ a pound. What is the cost?

38. At \$1.86 a bushel, what is the value of a carload of 425 bushels of potatoes?

39. At \$1.88 a bushel, what is the value of a carload of 760 bushels of wheat?

40. A carload of 850 bushels of corn costs \$1088. What is the cost per bushel?

41. An Automobile Road Map gives the following distances:

Boston to Springfield	94.7 miles
Springfield to Pittsfield	56.0 miles
Pittsfield to Albany	36.1 miles
Albany to Utica	95.0 miles
Utica to Syracuse	61.8 miles
Syracuse to Buffalo	152.0 miles
Buffalo to Niagara Falls	25.9 miles

(*a*) Find the distance from Boston to Pittsfield.
(*b*) Find the distance from Boston to Albany.
(*c*) Find the distance from Boston to Syracuse.
(*d*) Find the distance from Boston to Buffalo.
(*e*) Find the distance from Boston to Niagara Falls.
(*f*) Find the distance from Boston to Utica.
(*g*) Find the distance from Springfield to Syracuse.
(*h*) Find the distance from Springfield to Buffalo.
(*i*) Find the distance from Albany to Buffalo.
(*j*) Estimating the expense of operating an automobile at 8¢ per mile, find the cost of trips between cities named in (*a*) to (*i*). If an automobile carries 7 persons from Boston to Niagara Falls and each pays his share of the cost of running the car, how much should each one pay?

42. A carton containing 75 apples costs \$3.00. What is the average cost of an apple? Before the grocer disposes of all these apples, $\frac{1}{5}$ are lost by decay. The apples are sold at 5 cents each. How much is gained?

43. A carton containing 50 apples costs \$1.50. What is the cost of an apple? If $\frac{1}{5}$ of these decay and the remainder are sold at 5 cents each, how much is the profit?

44. At a ball park there are 40,000 people; $\frac{4}{5}$ travel by trolley cars. Estimating 125 persons per car, how many cars are needed to carry this crowd? At 5 cents each what is the total amount of fares collected?

45. If the rest of the crowd at the ball park travel by automobile and each automobile averages 4 people, how many automobiles are needed? At 25 cents each for parking these automobiles, how much is received?

46. A's salary for each of the last six years has been: \$1800, \$1900, \$2340, \$2484, \$2628, \$2772. Find his average yearly salary for this term of years.

47. The heights of each of 8 boys in a class are as follows: 5.9 ft., 5.8 ft., 5.8 ft., 5.7 ft., 5.4 ft., 5.0 ft., 4.9 ft., 4.7 ft. Find the average height.

48. The earnings of a company for 1915 were \$41,117,487.28. This was \$559,509.99 more than in 1914. What were the earnings for 1914?

49. A company has 257 former employees on its pension roll. The average pension paid is \$44 per month. How much is paid per month by this company in pensions? How much per year?

50. A company paid \$111,384 to 138 dependents of its employees. What was the average amount paid to each dependent?

CHAPTER II

EQUATIONS—RATIOS

I. Equations

26. Equations. The following exercises, of which several are solved, will indicate sufficiently the methods to be used in solving simple equations.

EXERCISES

1. If an orange costs 4 cents, what do 5 oranges cost?

Ans. 5×4 cents or 20 cents.

2. If an orange costs n cents, what do 5 oranges cost?

Ans. $5\,n$ cents.

(The sign $\times$ between 5 and n is omitted.)

3. If one baseball weighs 8 ounces, what do 6 baseballs weigh?

4. If one baseball weighs b ounces, what do 6 baseballs weigh?

5. If a boy is paid 30 cents a day for carrying bundles, how much is he paid for 10 days' work? For $4\frac{1}{2}$ days' work?

6. If a boy is paid n cents a day for carrying bundles, how much is he paid for 10 days' work? For $4\frac{1}{2}$ days' work?

7. If tea is being sold at 60 cents a pound, how much will 3 lb. cost? $\frac{1}{2}$ lb.?

8. If tea is being sold at t cents a pound, how much will 3 lb. cost? $\frac{1}{2}$ lb.?

9. A train travels at a rate of 35 miles an hour. What does 3×35 represent? What does $4\frac{1}{2} \times 35$ represent?

10. A train travels at a rate of m miles an hour. What does $3\ m$ represent? What does $4\frac{1}{2}\ m$ represent?

11. Six National League baseballs weigh 54 ounces. What is the weight of one baseball?

SOLUTION. Six times the weight of one baseball = 54 ounces. If you let w stand for the weight of one baseball, you get $6\ w = 54$. The statement $6\ w = 54$ is an ***equation***. To solve the equation $6\ w = 54$ means to find the value of w that multiplied by 6 gives 54. That value is 9, for 6 times 9 = 54.

Ans. The weight of one baseball = 9 ounces.

The two members (or sides) of an equation are like a pair of scales balanced. If you change the weight in one

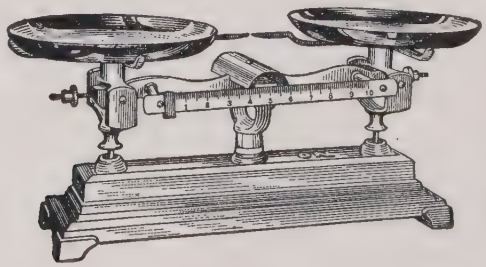

FIG. 1

scale pan, you must change the weight in the other scale pan by just the same amount, if the scales are to stay balanced. If you divide one member (or side) of an equation by 3, you must also divide the other member by 3.

12. A boy goes 18 miles on his bicycle in 3 hours; how many miles does he go in an hour?

SOLUTION. If you let m stand for the number of miles that he goes in an hour, the equation is

$$3\,m = 18.$$

To get m you divide 3 m by 3, and also divide 18 by 3:

$$m = 6.$$

Ans. The boy goes 6 miles per hour.

The solution of the equation $3\,m = 18$ may be arranged as follows:

$$3\,m = 18.$$

Dividing each member by 3,

$$m = 6.$$

13. A man gets a weekly salary of \$18. If he works six days, how much does he get per day?

SOLUTION. If d stands for the number of dollars he gets per day, the equation is

$$6\,d = 18.$$

Dividing each member by 6,

$$d = 3.$$

Each equation should be checked; that is, the number value of the letter should be put in the place of the letter in the first equation.

CHECK. In this example the check is $6 \times 3 = 18$.

Ans. The man gets \$3 per day.

14. One dozen Reach baseballs costs \$15. What is the price of one ball?

SOLUTION. Let p = the number of dollars that one ball costs, then $12\,p = 15$.

Dividing each member by 12,

$$p = 1\tfrac{1}{4}.$$

CHECK. $12 \times 1\tfrac{1}{4} = 15$.

Ans. The price of one baseball = \$1.25.

15. A battleship covering 675 miles in 30 hours travels how many miles per hour?

SOLUTION. Let m = number of miles per hour, then

$$30\ m = 675.$$

Dividing each member by 30,

$$m = 22\tfrac{1}{2}.$$

CHECK. $30 \times 22\tfrac{1}{2} = 675.$

Ans. The battleship travels $22\tfrac{1}{2}$ miles per hour.

(This means that $22\tfrac{1}{2}$ miles is the average rate per hour of the battleship. Its rate for the first hour might be less than $22\tfrac{1}{2}$ miles, for the second more than $22\tfrac{1}{2}$ miles.)

16. Solve the equation $5\ a = 15$.

SOLUTION. The given equation is

$$5\ a = 15.$$

Dividing each member by 5,

$$a = 3.$$

CHECK. $5 \times 3 = 15.$ *Ans.* $a = 3.$

NOTE. 3 is called the ***root*** of the equation $5\ a = 15$.

17. Solve the equation $14 = 4\ y$.

SOLUTION. The given equation is

$$14 = 4\ y.$$

Dividing each member by 4,

$$3\tfrac{1}{2} = y.$$

CHECK. $14 = 4 \times 3\tfrac{1}{2}.$ *Ans.* $y = 3\tfrac{1}{2}.$

NOTE. $3\tfrac{1}{2}$ is the ***root*** of the equation $14 = 4\ y$.

18. Solve the equation $4 = 28\ m$.

SOLUTION. Dividing each member by 28,

$$\frac{4}{28} = \frac{1}{7} = m.$$

CHECK. $4 = 28 \times \frac{1}{7}$. *Ans.* $m = \frac{1}{7}$.

What is the root of the equation $4 = 28\ m$?

19. Solve the equation $6.8\ p = 3.4$

SOLUTION. Dividing each member by 6.8,

$$p = \frac{3.4}{6.8} = \frac{1}{2}.$$

CHECK. $6.8 \times \frac{1}{2} = 3.4$ *Ans.* $p = \frac{1}{2}$.

NOTE. In solving Exercises 11–19, we have used the following axiom.*

AXIOM. *If equal numbers are divided by the same number the quotients are equal.*

Solve and check the following equations:

20. $5\ a = 15$.

21. $2\ x = 28$.

22. $12 = 3\ y$.

23. $28 = 4\ m$.

24. $39 = 13\ h$.

25. $5\ m = 17$.

26. $20 = 5\ b$.

27. $5\ r = 47$.

28. $8\ a = 2$.

29. $6 = 24\ b$.

30. $27\ q = 9$.

31. $5.3\ x = 1.59$

32. $18\ k = 5.4$

33. $12.4\ c = 49.6$

34. $18.4\ r = 280.6$

35. $.65 = .13\ b$

36. $.035 = .7\ t$

37. $98 = 1.4\ n$

38. $54\ r = 65.61$

39. $3\frac{1}{2}\ h = 7$.

40. $2\frac{1}{2}\ a = 10$.

41. $3\frac{1}{2}\ k = 20$.

42. $12 = 2\frac{2}{5}\ x$.

43. $18 = 4\frac{1}{2}\ z$.

44. $4\frac{2}{3}\ x = 42$.

45. $7\frac{1}{4}\ a = 32\frac{5}{8}$.

46. $16\frac{1}{2}\ b = 49\frac{1}{2}$.

47. $25 = 3\frac{1}{8}\ m$.

48. $9\frac{1}{8}\ x = 146$.

49. $\frac{1}{5}\ x = 7$.

50. $\frac{1}{2}\ k = 7\frac{1}{2}$.

51. $19 = \frac{1}{3}\ x$.

52. $24 = \frac{2}{3}\ y$.

53. $\frac{3}{4}\ a = 12$.

54. $\frac{3}{8}\ b = 15$.

55. $35 = \frac{5}{16}\ c$.

* An axiom is the statement of an accepted truth.

By the same method as in exercises 1–15, form equations for each of the following problems, and solve the equations.

56. A man earns $700 on a job requiring 28 weeks. How much are his earnings per week?

57. An automobile goes 165 miles in 6 hours. What is the average rate in miles per hour?

58. Saving 75 cents a week, in how many weeks would you have enough money to buy a bicycle valued at $19.50?

59. A freight car was built to carry a load of 34,720 lb. How many tons of coal could it carry? (1 ton of coal weighs 2240 lb. at wholesale.)

60. An aviator traveled 633 miles in 6 hours. What was his average rate in miles per hour?

61. An express train goes from Boston, Mass., to Portland, Me., a distance of 108 miles, in three hours. What is its average rate per hour?

62. A jitney bus operator expects to take in not less than $6.50 per day at 5 cents a passenger. How many passengers, at least, must he have?

63. There are 2500 bricks in a load weighing 500 lb. What is the weight of each brick in tenths of a pound? In ounces?

64. The mileage deducted from a mileage book for a party of 24 in traveling from one town to another was 528 miles. How far apart were the towns?

65. A machinist earns $21.60 per week, working 8 hours a day for six days. How much does he get an hour?

66. If 51.6 tons of old iron were sold for $624.36, what was the price per ton? What was the price per pound? What would 1320 lb. bring?

II. Ratios

27. Ratios. The following set of exercises, some of which are solved, will illustrate the idea of ratios, and the use of equations in problems that involve ratios.

EXERCISES

1. Compare 12 inches with 4 inches; 4 inches with 12 inches; 15 pounds with 5 pounds; 5 pounds with 15 pounds. Express each of the above comparisons as a fraction and reduce the fraction.

2. Compare 8 with 12.

SOLUTION. $\frac{8}{12} = \frac{2}{3}$; hence 8 is $\frac{2}{3}$ of 12.

That is, 8 has the same relation to 12 as 2 has to 3.

NOTE. This comparison of 8 to 12 or relation of 8 to 12, expressed by the fraction $\frac{8}{12}$, is called a ***ratio***.

3. Compare 12 with 8.

Here the comparison is expressed by the ratio $\frac{12}{8}$ or $\frac{3}{2}$.

4. What is the ratio of 15 to 20? Of 20 to 15? Of 10 to 16? Of 16 to 10?

5. Express the following ratios, reducing each to lowest terms: 18 to 10, 12 to 24, 6 to 9, 9 to 6, 11 to 17.

6. Express the ratios of the following quantities, reducing each ratio to the lowest terms:

(*a*) 18 in. to 24 in.
(*b*) 20 lb. to 70 lb.
(*c*) 35 ft. to 15 ft.
(*d*) 30 yd. to 36 yd.
(*e*) 42 lb. to 63 lb.
(*f*) 96 lb. to 108 lb.
(*g*) 48 da. to 60 da.
(*h*) 14 wk. to 49 wk.
(*i*) 12 oz. to 4 oz.
(*j*) 18 hr. to 9 hr.

(*k*) 4 oz. to 1 lb.
(*l*) 3 ft. to 5 yd.
(*m*) 3 yd. to 5 ft.
(*n*) 2 hr. to 45 min.
(*o*) 1760 ft. to 1 mi.
(*p*) 48 sq. in. to 1 sq. ft.
(*q*) $3\frac{1}{2}$ lb. to 8 oz.
(*r*) 144 cu. in. to 1 cu. ft.
(*s*) 75¢ to $3.25
(*t*) 5 qt. to 7 gal.

7. The ratio of a certain number to 6 is equal to $\frac{2}{3}$. Find the number.

SOLUTION. Let n = the number, then

$$\frac{n}{6} = \frac{2}{3}.$$

To solve the equation multiply each member by 6. The first member is

$$\frac{6n}{6} \text{ or } n.$$

The second member is

$$\frac{12}{3} \text{ or } 4.$$

Hence $n = 4.$

CHECK. $\frac{4}{6} = \frac{2}{3}.$

Ans. The number = 4.

8. The length of a room is 16 ft. The ratio of the width to the length is equal to $\frac{3}{4}$. Find the width of the room.

SOLUTION. Let w = width of room, then

$$\frac{w}{16} = \frac{3}{4}.$$

Multiplying each member by 16, the lowest common denominator of the two fractions, the first member is

$$\frac{16w}{16} \text{ or } w.$$

The second member is

$$\frac{48}{4} \text{ or } 12.$$

Hence $w = 12.$

CHECK. $\frac{12}{16} = \frac{3}{4}.$

Ans. The width is 12 ft.

9. What number has the same ratio to 25 that 16 has to 100?

SOLUTION. Let n = the number, then

$$\frac{n}{25} = \frac{16}{100}.$$

Multiplying each member by 100,

$$4\,n = 16.$$

Hence $n = 4.$

CHECK. $\frac{4}{25} = \frac{16}{100}.$

Ans. The number = 4.

10. Solve the equation $\frac{2\,n}{3} = 4.$

SOLUTION. The given equation is

$$\frac{2\,n}{3} = 4.$$

Multiplying each member by 3,

$$2\,n = 12.$$

Dividing each member by 2,

$$n = 6.$$

CHECK. $\frac{2 \times 6}{3} \stackrel{?}{=} 4.$

$$\frac{12}{3} = 4.$$

Ans. $n = 6.$

NOTE. In checking, each member must be simplified by itself.

11. Solve the equation $2 = \frac{4n}{5}$.

SOLUTION. The given equation is

$$2 = \frac{4n}{5}.$$

Multiplying each member by 5,

$$10 = 4n.$$

Dividing each member by 4,

$$2\tfrac{1}{2} = n.$$

CHECK. $$2 \stackrel{?}{=} \frac{4 \times 2\frac{1}{2}}{5}.$$

$$2 = \frac{10}{5}.$$

Ans. $n = 2\frac{1}{2}$.

12. Solve the equation $\frac{2n}{3} = \frac{4}{5}$.

SOLUTION. The given equation is

$$\frac{2n}{3} = \frac{4}{5}.$$

Multiplying each member by 15,

$$10n = 12.$$

Dividing each member by 10,

$$n = 1.2$$

CHECK. $$\frac{2 \times 1.2}{3} \stackrel{?}{=} \frac{4}{5}.$$

$$\frac{2.4}{3} \stackrel{?}{=} \frac{4}{5}.$$

$$.8 = .8$$

Ans. $n = 1.2$

(In checking, each member is simplified by itself.)

13. Solve the equation $\frac{r}{12} = \frac{3}{50}$.

SOLUTION. The given equation is

$$\frac{r}{12} = \frac{3}{50}.$$

Multiplying each member by 300,

$$25r = 18.$$

Dividing each member by 25,

$$r = .72$$

CHECK. $\frac{.72}{12} \stackrel{?}{=} \frac{3}{50}.$

$$.06 = .06$$

Ans. $r = .72$

NOTE. In solving Exercises 7–13, we have used the following axiom.

AXIOM. *If equal numbers are multiplied by the same number, the products are equal.*

Solve and check each of the following equations:

14. $\frac{y}{4} = \frac{11}{2}$.

15. $\frac{m}{4} = \frac{5}{2}$.

16. $\frac{a}{9} = \frac{7}{3}$.

17. $\frac{m}{10} = \frac{7}{15}$.

18. $\frac{p}{6} = \frac{5}{12}$.

19. $\frac{3}{4} = \frac{p}{3}$.

20. $\frac{r}{3} = \frac{5}{3}$.

21. $\frac{5r}{12} = \frac{5}{3}$.

22. $\frac{3b}{5} = \frac{2}{15}$.

23. $\frac{5}{6} = \frac{2w}{15}$.

24. $\frac{m}{5} = 7$.

25. $\frac{3m}{11} = 4$.

26. $\frac{5a}{2} = 8$.

27. $\frac{3b}{2} = .12$

28. $\frac{5x}{4} = .4$

29. $.25 = \frac{m}{4}$.

30. $\frac{7}{100} = \frac{m}{6}$.

31. $\frac{d}{4} = \frac{3}{100}$.

32. $\frac{3a}{4} = .09$

33. $\frac{5b}{12} = .2$

34. $\frac{4.2m}{3.6} = 2.1$

35. Out of a class of 50 pupils 40 were present. What was the number, or rate, per hundred present?

SOLUTION. This might be stated as follows. The ratio of 40 to 50 is equal to the ratio of what number to 100?

This statement would be expressed by the equation

$$\frac{40}{50} = \frac{n}{100},$$

where n = the number, or rate, per hundred, then

$$\frac{40}{50} = \frac{n}{100}.$$

Multiplying each member by 100,

$$80 = n.$$

CHECK. $$\frac{40}{50} \stackrel{?}{=} \frac{80}{100} \text{ or } .8 = .8$$

Ans. The rate per hundred is 80.

36. A boy pays 4 cents for 40 marbles. What is the price for 100?

SOLUTION. Let p = the price per 100, then

$$\frac{4}{40} = \frac{p}{100},$$

or

$$\frac{1}{10} = \frac{p}{100}.$$

Multiplying each member by 100,

$$10 = p.$$

CHECK. $$\frac{4}{40} \stackrel{?}{=} \frac{10}{100} \text{ or } .1 = .1$$

Ans. The price per 100 is 10 cents.

37. In 1910 the population of a town was 2400. It increased 600 during the next 5 years. What was the rate of increase per hundred for the five-year period?

38. One team in a baseball league has won 14 games out of 20 played. What is the team's rate of winning per hundred games?

39. In a certain class 9 out of every 10 pupils were present. The whole class numbered 70. How many were present?

SOLUTION. Let n = number of pupils present.

The ratio of 9 to 10 = the ratio of n to 70.

The equation is

$$\frac{9}{10} = \frac{n}{70}.$$

Multiplying each member by 70,

$$63 = n.$$

CHECK. $\frac{9}{10} \stackrel{?}{=} \frac{63}{70}$ or $.9 = .9$

Ans. 63 pupils were present.

40. At 45 cents a dozen how much will 16 oranges cost? (The ratio of 16 to 12 = ratio of what number to 45?) Form the equation and solve it.

41. Bananas are selling 15 for 25 cents. How much are they per dozen? (Be sure to arrange your statement so that the required number is the *first* number of one ratio.)

42. Lemons are selling 3 for 5 cents. How much will $1\frac{1}{2}$ dozen cost?

43. Tumblers are selling at 40 cents a dozen. How much will 9 tumblers cost?

44. If 4 apples cost 5 cents, what will 10 apples cost?

45. If soap is selling at 6 bars for a quarter, how much will 4 bars cost?

46. Post cards are sold at 80 for \$1.00. How much will 25 cost?

47. American League baseballs are sold at \$15.00 a dozen. Find the cost of 5 balls.

48. If 25 yards of cloth cost \$7.50, how many yards can be bought for \$69?

49. A school contains 2025 pupils; 4 out of every 9 are foreign born. How many pupils are foreign born?

50. If an aëroplane flies 65 miles in $2\frac{1}{2}$ hours, how far will it fly in 4 hours?

51. On a certain map a line $1\frac{1}{2}''$ long represents a distance of 120 miles. What distance is represented by a line 5″ long?

52. At \$7.75 a hundred pounds, find the cost of 275 pounds of lamb.

53. If 50 feet of garden hose sell at \$5.75, what will 125 feet cost?

54. At \$7.50 a ton of 2000 lb., what will 7000 lb. of coal cost?

55. The ratio $\frac{3}{4}$ is what rate per hundred?

SOLUTION.

$$\frac{3}{4} = \frac{x}{100}.$$

Multiplying each member by 100,

$$75 = x.$$

CHECK.

$$\frac{3}{4} = \frac{75}{100}.$$

Ans. $\frac{75}{100}$.

56. The ratio $\frac{3}{8}$ is what rate per hundred?

57. $\frac{1}{8}$ = how many per hundred?

58. $\frac{7}{8}$ = how many per hundred?

59. $\frac{5}{8}$ = how many per hundred?

60. $\frac{1}{3}$ = how many per hundred?

61. $\frac{2}{3}$ = how many per hundred?

62. $\frac{1}{6}$ = how many per hundred?

63. Copy and fill in the following table:

Ratio	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{7}{8}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{5}{6}$
Number per hundred	50										

64. Copy and fill in the following table:

Common fraction	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{7}{8}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{5}{6}$
Decimal fraction expressed in hundredths				$.12\frac{1}{2}$							

CHAPTER III

MEASUREMENT

I. Lines and Angles

28. Lines and Planes. Many of the surfaces about you are flat; for example, the blackboards, the top of your desk, the floor, the walls. These surfaces are called ***plane surfaces*** or ***planes***. If you draw a line on one of these surfaces with the aid of your ruler you say the line is straight, which means that it has the same direction, however long you make it.

Table of Linear Measure

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
$5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet = 1 rod (rd.)
320 rods or 5280 feet = 1 mile (mi.)

EXERCISES

[For the work that follows the pupil should have a ruler graduated in sixteenths of an inch on one edge and tenths of an inch on another.]

1. To test the straightness of the edge of your ruler, mark two points on a piece of paper. Using your ruler draw a line through them. Turn the ruler over and draw a line through the same two points along the same edge. If the ruler has a straight edge, would the two lines be different lines or the same line? [For this and other constructions your pencil should be sharpened to a fine point.]

2. How many points are necessary to show the direction of a straight line?

3. How may a straight-edged ruler be used to test whether a surface is plane or not? [Place the edge of your ruler against the surface to be tested. If the edge touches the surface at all points for every possible position of the ruler, the surface is plane.] Test the top of your desk, the blackboards, and other objects that you think are plane.

4. Draw a straight line $\frac{5}{8}$ of an inch long. Compare it with a line .6 of an inch long. With a line .7 of an inch long.

5. Draw a straight line $1\frac{7}{8}$ inches long. Compare it with a line 1.8 inches long. With a line 1.9 inches long.

6. Draw four straight lines of different lengths. Measure each to the nearest tenth of an inch. Add the lengths and divide the sum by 4 to get the average length of the four lines.

7. Draw five straight lines of different lengths. Measure each to the nearest tenth of an inch. Find the average length of the five lines by adding them and dividing by 5.

8. Draw three straight lines and measure as above. Find the average length.

9. Measure the two straight lines in Fig. 2 to the nearest tenth of an inch. Find the ratio of the two lines. Their lengths are 1.2 in., and 1.8 in., hence their ratio is

Fig. 2

$$\frac{1.2}{1.8}, \text{ or } \frac{12}{18}, \text{ or } \frac{2}{3}.$$

10. Draw two straight lines of different lengths. Measure them to the nearest tenth of an inch. Find their ratio.

11. Draw two other straight lines and find their ratio as above.

12. Draw a straight line. Measure it to the nearest eighth of an inch. Measure it to the nearest tenth of an inch. Change your measurement in eighths to a decimal and compare it with the measurement in tenths.

[Henceforth, the word *line* will stand for *straight line.*]

29. ***Angles.*** When a straight line starting from a point is turned about that point from one position to another, an ***angle*** ($\angle$) is formed by the two positions of the line. The greater the amount of turning the greater the angle. For example, the hands of a clock form different angles with each other as they move from minute to minute.

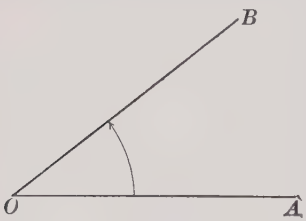

FIG. 3

30. In Fig. 3 the line is turned about O from the position OA to the position OB, forming the angle AOB. O is called the ***vertex*** of the angle AOB, and OA and OB the ***sides***. In reading an angle the vertex letter is always read in the middle ($\angle\ AOB$).

31. If the line is turned about O from the position OA until it again takes the position OA, a ***revolution*** is completed. A revolution is divided into 360 equal parts called ***degrees*** (360°). A degree is divided into 60 equal parts called ***minutes*** (60′). A minute is divided into 60 equal parts called ***seconds*** (60″).

TABLE OF DEGREE MEASURE

$60'' = 1$ minute

$60' = 1$ degree

$360° = 1$ revolution

32. The hands of a clock at 6 o'clock form an angle equal to one half a revolution, or 180°. One instrument used for measuring angles is called a ***protractor***. (See Fig. 4.) For the following exercises the student should have this or some other form of protractor.

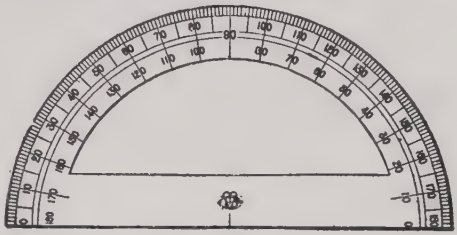

FIG. 4

EXERCISES

1. With the ruler draw an angle of 90°. Test the accuracy of it with your protractor.

2. Draw angles of 60°, 30°, 120°, 150°, 45°, 80°, 140°. Test the accuracy of each with your protractor. Practice drawing without your protractor until you become fairly accurate.

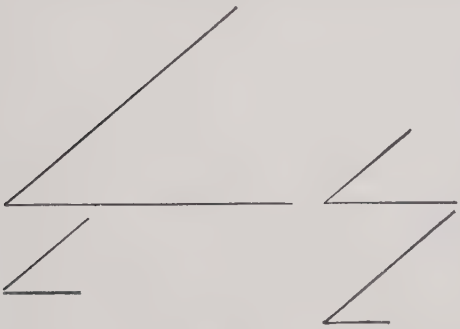

FIG. 5

3. Measure the four angles in Fig. 5 with your protractor. Does the size of the angle depend upon the lengths of its sides? Upon what does the size of the angle depend?

33. A ***right angle*** is one fourth of a revolution, or an angle of 90°. At 9 o'clock and at 3 o'clock the hands of a clock are at right angles to each other.

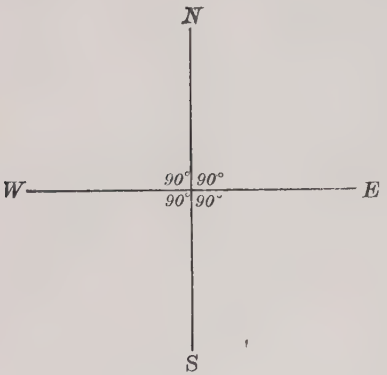

Fig. 6

34. ***Perpendicular lines*** (⊥) are lines that form right angles with each other. The East-West lines and the North-South lines are perpendicular to each other.

35. ***Parallel lines*** (||) are lines that do not meet, however far they are extended.

EXERCISES

1. Place lines in different positions, as in Fig. 7, and draw a perpendicular to each of them by means of a protractor.

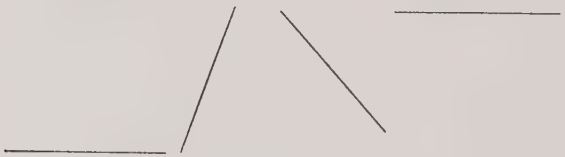
Fig. 7

2. Draw a line, then two perpendiculars to the line at two points. What do you note about these perpendiculars, no matter how long you make them? What name do you give to such lines?

3. Draw two parallel lines. Draw a third line across them and letter as in Fig. 8. Compare the different angles formed, using your protractor. What angles seem to be equal to $\angle AOB$? What angles seem to be equal to $\angle AOE$? $\angle AOB$ and $\angle BOC$ together

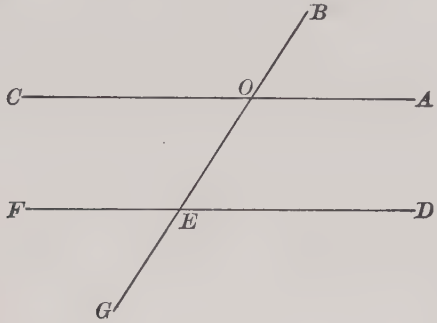

Fig. 8

form an angle of how many degrees? $\angle AOB$ and $\angle AOE$ together form an angle of how many degrees? Name other pairs of angles that are related to each other in the same way.

These pairs of angles are called ***supplementary angles***, or ***supplements*** of each other. Give a definition for supplementary angles.

4. How many degrees are there in the supplement of 40°? 110°? 65°? 42°? 137°?

5. How many degrees and minutes are there in the supplement of 30° 20′? 138° 15′? 64° 17′? 84° 42′? 122° 47′?

Specimen Solution. $180°$

$$\underline{30° \; 20'}$$

$$149° \; 40'$$

6. How many degrees, minutes, and seconds are there in the supplement of 4° 15′ 30″? 72° 42′ 20″? 115° 37′ 13″? 12° 44′ 45″? 153° 17′ 35″?

$$\begin{array}{r} 180^\circ \\ 4^\circ\ 15'\ 30'' \\ \hline 175^\circ\ 44'\ 30'' \end{array}$$

7. An acute angle is an angle less than 90°. Write a list of acute angles found in Exercises 4, 5, and 6.

8. An obtuse angle is an angle greater than 90° but less than 180°. Write a list of obtuse angles found in Exercises 4, 5, and 6.

9. What kind of an angle do the hands of a clock make at 2.30? At 4.00? At 3.00? At 8.50? At 6.25? At 7.05? At 10.40? At 1.10? At 6.00? At 9.00?

10. Draw three acute angles of different sizes. Measure them; find their sum.

11. Draw three obtuse angles of different sizes. Measure them; find their sum.

12. Draw one acute angle and one obtuse angle. Measure them. Find the ratio of the acute angle to the obtuse angle.

13. Draw two acute angles of different sizes. Measure them. Find the ratio of the larger angle to the smaller.

14. Draw an acute angle. Measure it. Find its ratio to a right angle.

15. Draw an obtuse angle. Measure it. Find its ratio to a right angle.

16. Draw two parallel lines and a third line across them. Measure two of the supplementary angles. Find the ratio of the smaller to the larger.

II. Plane Figures

36. A ***plane figure*** is a figure which has all of its lines and points lying on a flat surface, or plane.

37. A ***square*** is a plane figure inclosed by four lines of equal length drawn at right angles to each other.

In the square $ABCD$, $AB = BC = CD = DA$. Angles A, B, C, and D are right angles. AB, BC, CD, DA, are called the ***sides*** of the square. Points A, B, C, and D are called the ***vertices*** (*plural of* vertex).

Fig. 9

EXERCISES

1. Measure the sides of the square $ABCD$ to the nearest tenth of an inch. What is the sum of the four sides?

The sum of the sides is called the ***perimeter*** of the square.

2. Draw a square having each side equal to 1.3″. What is its perimeter?

[The symbol (′) is commonly used to express feet and (″) to express inches.]

3. Draw a square whose side is 1.7″. What is its perimeter?

4. Draw a square whose side is $2\frac{1}{8}$″. What is its perimeter?

5. Draw a square whose perimeter is 6.4″.

6. Draw a square whose perimeter is 5.2″.

7. Draw to the scale of 10 to 1 a square having each side 15″ long. This means that a line 1″ long is drawn to represent a line 10″ long. How long should the line be to represent a 15″ line?

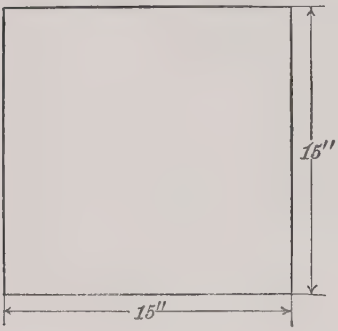

FIG. 10

8. Draw to the scale of 20 to 1 a square having each side equal to 30″.

9. Draw to the scale of 10 to 1 a square having each side equal to 17″. In your figure, 17″ should be represented by 1.7″. Why?

10. Draw to the scale of 10 to 1 a square whose perimeter is 48″.

11. Draw to the scale of 5′ to 1″ a square having each side equal to 12′. Here 1″ represents 5′, hence 2.4″ will represent 12′ (since 12 contains 5, 2.4 times).

12. Draw to the scale of 10′ to 1″ a square having each side equal to 18′.

13. A square room is 17′ on each side. Draw a plan of it to the scale of 10′ to 1″.

14. A square lot of land is 120′ on each side. Draw a plan of the lot to the scale of 100′ to 1″.

15. The distance around a square block is 440′. Draw a plan of the block to the scale of 100′ to 1″.

16. Draw to the scale of 40′ to 1″ the plan of a baseball diamond. A baseball diamond is a square 90 feet long. Mark the vertices 1, 2, 3, and 4 for bases, 4 being "Home." Measure the distance on your plan from second base to Home to the nearest tenth of an inch. How many feet does this measurement represent?

38. A ***rectangle*** is a plane figure inclosed by two pairs of parallel lines at right angles to each other.

In the rectangle $ABCD$, the side AB is called the ***base***, and the side AD, which is perpendicular to the base, is called the ***height*** (or ***altitude***) of the rectangle.

A square is a special kind of rectangle.

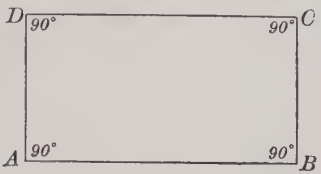

FIG. 11

EXERCISES

1. Measure the sides of the rectangle $ABCD$ to the nearest tenth of an inch. (*a*) What is its perimeter? (*b*) What is the ratio of the base to the height?

2. Draw a rectangle whose base is 3.2″ and height 1.4″, using ruler and protractor. (*a*) What is its perimeter? (*b*) What is the ratio of the height to the base?

3. Draw a rectangle whose base is $2\frac{1}{4}$″ and whose height is $1\frac{7}{8}$″. (*a*) What is its perimeter? (*b*) What is the ratio of the base to the height?

4. Draw to the scale of 10 to 1 a rectangle whose base is 30″ and whose height is 20″.

5. Draw to the scale of 10′ to 1″ a rectangle whose base is 32′ and whose height is 24′. (*a*) What is its perimeter? (*b*) What is the ratio of its altitude to its base?

6. The base of a rectangle is 2.1″ and its perimeter is 6.8″. Draw the rectangle.

7. Draw to the scale of 20′ to 1″ the plan of a rectangular lot of land 60′ long and 45′ wide.

8. Draw to the scale of 100′ to 1″ the plan of a rectangular garden plot 225′ long and 180′ wide.

9. Draw a rectangle $1\frac{3}{4}''$ long and 1.4″ wide. (*a*) What is its perimeter? (*b*) What is the ratio of its length to its width?

10. In the diagram of Fig. 12, the scale is 100′ to the

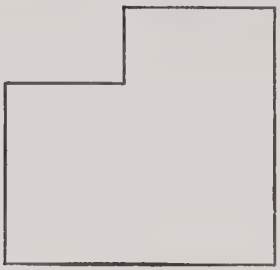

Fig. 12

inch and all the angles are right angles. Measure the sides to the nearest tenth of an inch. Find its perimeter.

11. Measure the length and width of your classroom to the nearest tenth of a foot. Draw a plan of it to the scale of 10′ to 1″.

12. If the school yard is rectangular, measure the length and width to the nearest foot. Draw a plan of it to the scale of 100′ to 1″.

39. A ***parallelogram*** (□) is a plane figure inclosed by two pairs of parallel lines.

In the parallelogram $ABCD$, Fig. 13, AB is called the ***base***, and h, perpendicular to the base, is called the ***height*** (or ***altitude***). In this parallelogram $ABCD$, what kind of angles are the angles at B and D? At A and C?

If a parallelogram were so drawn that all the four angles were equal, what special name would you give the parallelogram?

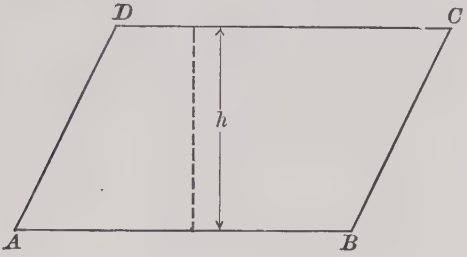

FIG. 13

EXERCISES

1. Measure the four sides of the parallelogram $ABCD$ to the nearest tenth of an inch.

(*a*) How do the sides AB and CD compare in length? AD and BC?

(*b*) What is the ratio of BC to AB?

(*c*) Measure its height and find the ratio of the height to the base.

(*d*) What is the perimeter?

(*e*) Using your protractor compare the opposite angles A and C; also $\angle B$ with $\angle D$.

(*f*) Add the number of degrees in the four angles.

(*g*) $\angle A + \angle B = (?)$ $\angle B + \angle C = (?)$

(*h*) $\angle A$ and $\angle B$ are supplementary. Why?

(*i*) If the figure were a rectangle, to what would the ratio of the height to the base be equal? (h unchanged.)

2. Draw another parallelogram (not a rectangle). Find the results as required in exercise 1.

3. By referring to the results obtained in exercises 1 and 2, complete the following statements:

(*a*) The opposite sides of a parallelogram are (?)

(*b*) The opposite angles of a parallelogram are (?)

(*c*) The two angles at the base of a parallelogram are (?)

(*d*) The sum of the four angles of a parallelogram is equal to (?)

4. The base of a parallelogram is $2\frac{1}{2}''$ long. One angle at the base is 50°.

(*a*) How large is the other angle at the base?

(*b*) Another side of the parallelogram is $1\frac{3}{8}''$ long. Have you enough parts given so that you can draw the whole parallelogram?

(*c*) With your ruler and protractor draw it, if possible.

5. Two of the sides of a parallelogram are 3.2″ and 1.7″; these sides include an angle of 45°. How many degrees in the angle at the other end of the 3.2 inch side? Using your ruler and protractor draw the parallelogram.

6. Two of the sides of a parallelogram are $2\frac{3}{8}''$ and $1\frac{7}{8}''$; these sides include an angle of 55°. Draw the parallelogram.

7. Two of the sides of a parallelogram are 24″ and 18″; these sides include an angle of 40°. Draw the parallelogram to the scale of 10 to 1. The sides of the parallelogram drawn to scale will be 2.4″ and 1.8″ but the angle will remain 40°. In drawing to scale the angles will not change because the angles fix the direction of the sides.

8. Two of the sides of a parallelogram are 40″ and 30″; these sides include an angle of 65°. Draw the parallelogram to the scale of 20 to 1.

9. Two of the sides of a parallelogram are 50′ and 15′; the included angle is 75°. Draw the parallelogram to the scale of 20′ to 1″.

10. Two of the sides of a parallelogram are 120′ and 90′; the included angle is 115°. Draw the parallelogram to the scale of 100′ to 1″.

40. A ***triangle*** (△) is a plane figure inclosed by three straight lines.

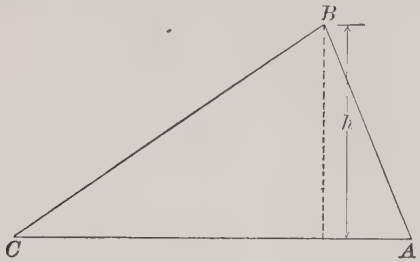

FIG. 14

In the triangle ABC, AC is called the *base*, and h, perpendicular to the base, is called the ***height*** (or ***altitude***).

EXERCISES

1. With your ruler measure the three sides and the height of triangle ABC to the nearest tenth of an inch.

(*a*) What is the perimeter?

(*b*) What is the ratio of AB to AC?

(*c*) What is the ratio of BC to AC?

(*d*) What is the ratio of the height to the base?

2. With your protractor measure the three angles of the triangle ABC.

(*a*) What kind of angle is each, right, acute, or obtuse?

(*b*) What is the ratio of $\angle A$ to $\angle C$?

SOLUTION. By measurement,

$$\angle A = 68°, \angle C = 35°,$$

$$\text{then } \frac{\angle A}{\angle C} = \frac{68}{35}$$

Ans. The ratio of $\angle A$ to $\angle C = 68$ to 35.

(*c*) Find the sum of the three angles.

3. With your ruler draw a triangle having all its sides of different lengths. Draw the height.

(*a*) Measure the three sides and the height to the nearest tenth of an inch. What is the perimeter of the triangle?

(*b*) Measure the three angles. The sum of the three angles = (?)°.

4. Two sides of a triangle are 2.4″ and 1.8″ long; these sides include an angle of 50°. Draw the triangle. Measure the other two angles. Find the sum of the three angles.

5. Two sides of a triangle are $2\frac{1}{4}$″ and $\frac{7}{8}$″; these sides include an angle of 42°. Draw the triangle.

(*a*) Measure the third side. Find the perimeter of the triangle.

(*b*) Measure the other two angles. Find the sum of the three angles.

6. Two sides of a triangle are 23″ and 17″; these sides include an angle of 68°. Draw the triangle to the scale of 10 to 1.

(*a*) Find the perimeter of the triangle.

(*b*) Measure the other two angles. Find the sum of the three angles.

7. A grass plot is triangular. Two of the sides are each 120′ in length; they include an angle of 65°. Draw the plan of the plot to the scale of 100′ to 1″.

(*a*) Find its perimeter.

(*b*) Measure the other two angles. Find the sum of the three angles.

8. This plan, Fig. 15, is drawn to the scale of 10′ to 1″.

What fractional part is the height of the base? How many hundredths of it?

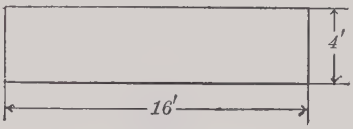

FIG. 15

9. The height of a rectangle is 5′ and the base is 25′. What fractional part is the height of the base? How many hundredths of it?

10. Copy the following table and fill in the required results, where h denotes the height of a rectangle and b denotes the base of the rectangle.

h	b	$\frac{h}{b}$	HUNDREDTHS
6′	12′	$\frac{1}{2}$	.50
3′	9′		
12″	48″		
6″	30″		
7″	42″		
24′	32′		
1.8″	2.7″		
15′	18′		
1.2′	3.2′		
1′3″	2′		
1′2″	1′4″		
7″	7′		

III. Representation of Number Data by Line-graphs

41. Line measurements are used frequently to represent the amounts of other quantities. The following exercises, several of which are explained in detail, make clear how this is done.

EXERCISES

1. The average length of the school year (1914) for certain states and the United States is represented by the lines in Fig. 16, where 1″ represents 100 days. Measure the lines to the nearest hundredth of an inch. To get the nearest hundredth, think the tenth of the inch as divided into ten equal divisions. These divisions are very small. From the measurements write the average number of days in the school year.

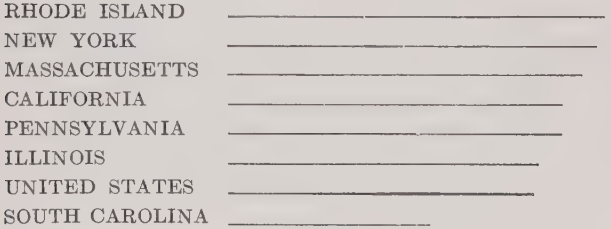

Fig. 16

The length of line for Massachusetts is 1.84″. Since 1″ represents 100 days, Massachusetts has 184 days for the average number of days in its school year.

The line for New York is 1.90″ long, hence New York has (?) days.

2. The average number of people per square mile in 1915 is represented by the line-graph of Fig. 17, where 1″ represents 100 people. Measure each of the following

lines, and write the number of people per square mile for each one.

FIG. 17

3. Miles of electric railway tracks (1915) were distributed as represented by the following line-graphs, where 1″ represents 5000 miles.

NEW ENGLAND STATES $1.22 \times 5000 = 6100$ *miles*

EASTERN STATES

CENTRAL STATES

SOUTHERN STATES

WESTERN STATES

FIG. 18

These numbers are not the exact numbers, but they are as near the exact numbers as you can get from the measurements made. Such numbers are often called "round" numbers (and sometimes "approximate" numbers).

4. The number of miles of railway tracks (1914) for certain states was as follows:

Texas	15,758
Illinois	12,140
Pennsylvania	11,634
Iowa	9,917
New York	8,511
Massachusetts	2,130

Make line-graphs for these, letting 1″ represent 5000 miles.

Texas: $\frac{15{,}758}{5{,}000} = 3.15$ to the nearest hundredth.

TEXAS 3.15″ long

FIG. 19

To get the nearest hundredth find out, by dividing, what the figure in the *thousandths'* place is. If that figure is 5 or more, the *nearest* hundredth is 1 higher than you get by dividing; if that figure in the thousandths' place is less than 5, the figure in the hundredths' place is not changed.

Illinois: $\frac{12,140}{5,000} = 2.43.$ Iowa: $\frac{9,917}{5,000} = 1.98.$

Pennsylvania: $\frac{11,634}{5,000} = 2.33.$ New York: $\frac{8,511}{5,000} = 1.70.$

Massachusetts: $\frac{2,130}{5,000} = 0.43.$

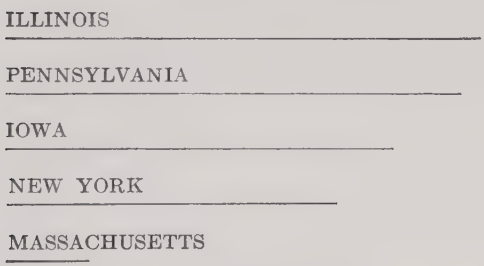

FIG. 20

5. Represent the lengths of the following rivers by line-graphs, letting 1″ represent 1000 miles.

Amazon	3,800 miles	Niger	2,900 miles
Amur	2,600 miles	Nile	3,766 miles
Congo	3,000 miles	Rhine	700 miles
Danube	1,725 miles	Rio Grande	1,800 miles
Hoangho	2,600 miles	St. Lawrence	2,150 miles
La Plata	2,300 miles	Volga	2,300 miles
Mackenzie	2,300 miles	Yangtse	3,400 miles
Mississippi-Missouri	4,200 miles	Yenisei	3,300 miles

6. Represent the heights of the following buildings in New York City by *vertical* line-graphs, letting 1″ represent 200 feet.

Woolworth	750′
Metropolitan	700′
Singer	612′
Equitable	486′
Times	420′
Flatiron (Fuller)	286′

7. Represent the heights of the following mountains by *vertical* line-graphs, letting 1″ represent 5000 feet.

Mt. McKinley	20,300′	Mt. Everest	29,002′
Mont Blanc	15,782′	Mt. Aconcagua	23,080′

8. Represent the number of people speaking the following languages (1910) by horizontal line-graphs, letting 1″ represent 50,000,000 people.

English	160,000,000
German	130,000,000
French	70,000,000
Russian	100,000,000
Italian	50,000,000

9. Represent by horizontal line-graphs the relative sizes of the following islands, using different scales for the two groups.

Group I		Group II	
Bahamas	5,400 sq. mi.	Borneo	284,000 sq. mi.
Corsica	3,400 sq. mi.	Greenland	827,300 sq. mi.
Crete	2,900 sq. mi.	Japan	160,000 sq. mi.
Hawaiian	6,450 sq. mi.	Madagascar	227,000 sq. mi.
Jamaica	4,200 sq. mi.	New Guinea	330,000 sq. mi.
Long Is.	1,376 sq. mi.	Philippines	115,026 sq. mi.

CHAPTER IV

PERCENTAGE — DISCOUNT — INTEREST

I. Percentage

42. ***Per cent*** means the number per hundred. Thus 96 per cent (written 96%) means 96 per hundred. For example, 96% attendance means that 96 of every 100 pupils are present.

Example 1. In a spelling test of 100 words,

John spelled 96 correctly, or 96 per 100, or 96%;
Mary spelled 92 correctly, or 92 per 100, or 92%;
James spelled 88 correctly, or 88 per 100, or 88%;
Henry spelled 84 correctly, or 84 per 100, or 84%.

As there were 100 words in this test and per cent means per hundred, the number of words spelled correctly by each pupil gives also the per cent obtained by that pupil.

Example 2. In a spelling test of 25 words,

John spelled 24 correctly, or $\frac{24}{25}$ or $\frac{96}{100}$ or 96%;
Mary spelled 23 correctly, or $\frac{23}{25}$ or $\frac{92}{100}$ or 92%;
James spelled 22 correctly, or $\frac{22}{25}$ or $\frac{88}{100}$ or 88%;
Henry spelled 21 correctly, or $\frac{21}{25}$ or $\frac{84}{100}$ or 84%.

John spelled correctly at the rate of 24 per 25 or 96 per 100; that is, John spelled 96% of the words correctly.

Mary spelled correctly 23 of 25 (or at the rate of 92 per 100) and so spelled 92% (of the words) correctly.

James spelled correctly 22 of 25 (or 88 per 100) or 88%.

Henry spelled correctly 21 of 25 (or 84 per 100) or 84%.

EXERCISES

1. If 20 words were given, and

John spelled correctly 18 words,
Mary spelled correctly 19 words,
James spelled correctly 15 words,
Henry spelled correctly 14 words,

then what per cent of the words did each spell correctly?

What per cent of the words did each misspell?

(John missed 2 out of 20 words or at the rate of 10 per 100 or 10%.)

2. Ten examples were given in a speed test and only correct answers counted. John had 6 correct, Mary 9, James 7, Henry 8. What per cent did each obtain on this test?

3. In a series of 5 games, Team A won 3 games and Team B, 2 games. What per cent of the games did each win?

43. *Per cent* may also be called the number of hundredths of some given amount. For example, if a line AB

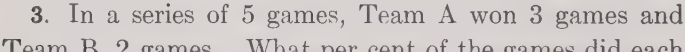

FIG. 21

of definite length be divided into 100 equal parts, as shown in Fig. 21, then

10 of these parts (10 of 100) represent 10% of the length of the line,

20 parts represent 20% of the line,

60 parts represent 60% of the line,

85 parts represent 85% of the line,

$33\frac{1}{3}$ parts represent $33\frac{1}{3}\%$, or one third, of the line.

44. If our foot rule is divided into ten equal parts (tenths), then one of these parts is $\frac{1}{10}$ of a foot or $\frac{10}{100}$ of a foot or 10% of a foot,

2 of these parts, $\frac{20}{100}$ of a foot or 20% of a foot,

4 of these parts, $\frac{40}{100}$ of a foot or 40% of a foot.

If our inch is subdivided into *tenths*, express the per cent that one tenth is of the inch. Do this for .2 in., .3 in., etc.

45. The relation that any line bears to a second line may be expressed as a per cent relation, for the ratio that the first line bears to the second may be expressed as a ratio per hundred.

Thus, if a line $AB = 4.5$ in., and a line $CD = 5.0$ in., then,

$$\frac{AB}{CD} = \frac{4.5}{5.0} = \frac{90}{100};$$

that is,

line AB is 90% of line CD.

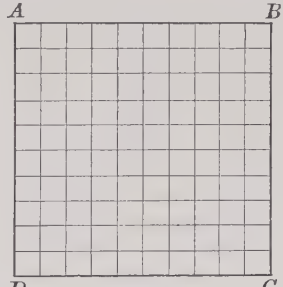

FIG. 22

46. If we have a square $ABCD$ subdivided into 100 equal squares as shown, then

1 of the small squares = $\frac{1}{100}$, or .01, or 1%, of the large square;

2 of the small squares = $\frac{2}{100}$, or .02, or 2%, of the large square;

6 of the small squares = .06, or 6%, of the large square;

a row of small squares = .10, or 10%, of the large square.

2 rows of small squares = what % of the large square?

3 rows of small squares = what % of the large square?

4 rows of small squares = what % of the large square?

8 rows of small squares = what % of the large square?

EXERCISES

1. What per cent of a foot is 6 in.? 4 in.? 9 in.? 12 in.?

2. What per cent of a yard is 18 in.? 9 in.? 27 in.? 36 in.?

3. What per cent of a square foot is 72 sq. in.? 36 sq. in.? 108 sq. in.?

4. The following table represents the results of a test in arithmetic. Fill in the values required.

Student	Number of Examples	Number Right	Number Wrong	Per Cent Right	Per Cent Wrong
a.	10	8	?	?	?
b.	10	7	?	?	?
c.	10	6	?	?	?
d.	10	5	?	?	?
e.	10	?	4	?	?
f.	10	?	3	?	?
g.	10	?	?	?	50
h.	10	?	?	30	?

5. The following table represents the results of a test in spelling. Fill in the values required.

Student	Number of Words	Number Correct	Number Incorrect	Per Cent Correct	Per Cent Incorrect
a.	20	17	?	?	?
b.	25	21	?	?	?
c.	50	?	7	?	?
d.	40	22	?	?	?
e.	40	?	16	?	?
f.	50	?	?	80	?
g.	25	?	?	60	?
h.	40	?	?	?	10

6. The following table represents the record of attendance at several schools. Fill in the values required.

School	Total Number of Pupils	Number Present	Number Absent	Per Cent of Attendance	Per Cent of Absence
a.	200	190	?	?	?
b.	300	285	?	?	?
c.	500	?	45	?	?
d.	800	?	72	?	?
e.	1000	850	?	?	?
f.	1500	1350	?	?	?

47. The original amount is commonly regarded as 100% (of itself). Thus, in Ex. 4 (*a*), to get 100% on the test would mean to get 10 examples right.

48. It is important to notice that per cent always refers to per cent (or number of hundredths) of *some definite amount.* Frequently the definite amount is not stated. For example, per cents obtained by students in tests such as, 95%, 92%, 85%, etc., do not necessarily tell us how many examples or questions were given. Such per cents, however, do give us a means of comparing the results.

EXERCISES

1. What per cent of 25 is 3? 12? 17? 23? 25? 28? 30? 50?

Specimen Solution. Make denominator of fraction 100.

3 = what fractional part of 25?

$3 = \frac{3}{25}$ of 25.

$\frac{3}{25} = \frac{12}{100}$.

Ans. 3 is 12% of 25.

2. What per cent of 50 is 10? 18? 26? 37? 50? 57? 86? 90?

3. What per cent of 300 is 27? 36? 54? 90? 129? 210? 300? 315?

4. What per cent of 400 is 28? 120? 260? 300? 380? 400? 800?

5. What per cent of 600 is 6? 36? 60? 72? 240? 300? 420? 600?

49. FRACTIONAL PARTS OF THE WHOLE EXPRESSED AS HUNDREDTHS, AND AS PER CENTS

$\frac{1}{10} = .10 = 10\%$	$\frac{3}{4} = .75 = 75\%$
$\frac{1}{8} = .12\frac{1}{2} = 12\frac{1}{2}\%$	$\frac{2}{3} = .66\frac{2}{3} = 66\frac{2}{3}\%$
$\frac{1}{6} = .16\frac{2}{3} = 16\frac{2}{3}\%$	$\frac{3}{8} = .37\frac{1}{2} = 37\frac{1}{2}\%$
$\frac{1}{5} = .20 = 20\%$	$\frac{5}{8} = .62\frac{1}{2} = 62\frac{1}{2}\%$
$\frac{1}{4} = .25 = 25\%$	$\frac{7}{8} = .87\frac{1}{2} = 87\frac{1}{2}\%$
$\frac{1}{3} = .33\frac{1}{3} = 33\frac{1}{3}\%$	$\frac{5}{6} = .83\frac{1}{3} = 83\frac{1}{3}\%$
$\frac{1}{2} = .50 = 50\%$	

The above fractions are so commonly used that their equivalent per cents should be memorized.

EXERCISES

1. What per cent of 8 is 6?

SOLUTION. Reduce fraction to lowest terms; then give its equivalent decimal.

6 = what fractional part of 8?

$6 = \frac{6}{8}$ of 8, or $\frac{3}{4}$ of 8.

$\frac{3}{4} = .75$ or 75%.

Ans. 6 is 75% of 8

2. What per cent of 24 is 20?

20 = what fractional part of 24?

$20 = \frac{20}{24}$ of 24.

$\frac{20}{24} = \frac{5}{6}$.

$\frac{5}{6} = .83\frac{1}{3}$ or $83\frac{1}{3}\%$.

Ans. 20 is $83\frac{1}{3}\%$ of 24.

3. In Fig. 23, the number at the center is what per cent of each number on the rim?

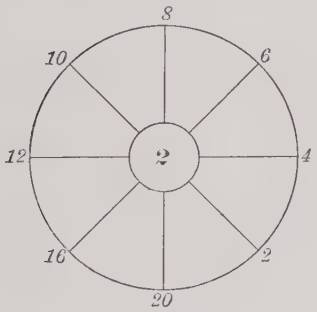

FIG. 23

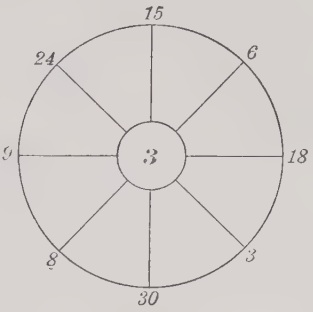

FIG. 24

Begin as follows:

2 is $\frac{1}{4}$ of 8 or 2 is 25% of 8.

2 is $\frac{1}{5}$ of 10 or 2 is 20% of 10.

In this manner use each number on the rim.

4. In Fig. 24, the number at the center is what per cent of each number on the rim?

5. In Fig. 25, the number at the center is what per cent of each number on the rim?

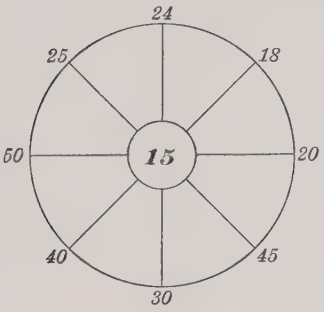

FIG. 25

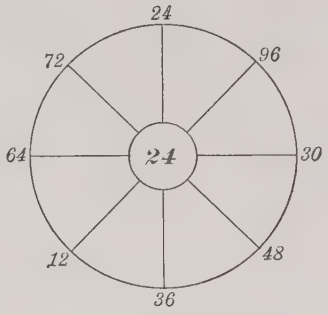

FIG. 26

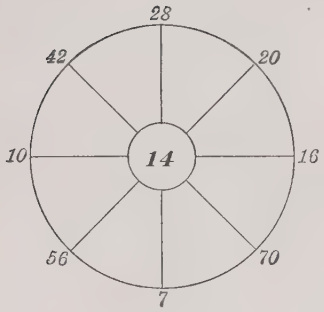

FIG. 27

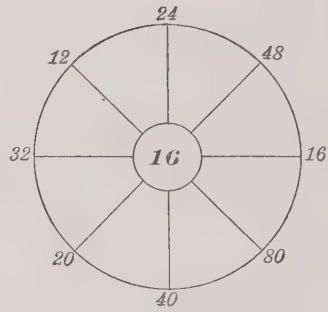

FIG. 28

6. In Fig. 26, the number at the center is what per cent of each number on the rim?

7. In Fig. 27, the number at the center is what per cent of each number on the rim?

8. In Fig. 28, the number at the center is what per cent of each number on the rim?

9. What per cent of 64 is 16? 32? 48? 8? 24? 40? 56? 64?

10. What per cent of 56 is 14? 28? 7? 21? 35? 56? 70? 84?

11. What per cent of 96 is 24? 32? 48? 64? 96? 120? 16? 80?

12. What per cent of 120 is 40? 48? 72? 96? 100? 140? 60? 240?

13. What per cent of 72 is 36? 18? 24? 27? 45? 54? 63? 72?

14. What per cent of 144 is 16? 48? 72? 108? 120? 132? 180? 288?

15. What per cent of 32 is 15?

SOLUTION. 15 = what fractional part of 32?

$$\begin{array}{r} .46\frac{7}{8} \\ 32\overline{)15.00} \\ \underline{128} \\ 220 \\ \underline{192} \\ 28 \end{array}$$

15 = $\frac{15}{32}$ of 32.

$\frac{15}{32}$ = what number of hundredths?

$\frac{15}{32}$ = $.46\frac{7}{8}$ or $46\frac{7}{8}\%$.

Ans. 15 is $46\frac{7}{8}\%$ of 32.

$\frac{28}{32} = \frac{7}{8}$

EXPLANATION. To express $\frac{15}{32}$ as a number of hundredths you must divide 15 by 32 as follows:

Put a decimal point after 15. Annex 2 zeros for hundredths. Divide as shown. Place a decimal point in answer over point in dividend.

Express the remainder, if any, as fractional part of divisor (this is a part of a hundredth).

16. What per cent of 32 is 22? 31? 32? 38? 58?

SOLUTION. $22 = \frac{22}{32}$ of 32.

$$16\overline{)11.00} = .68\tfrac{3}{4}$$

$$\begin{array}{r} 96 \\ \hline 140 \\ 128 \\ \hline 12 \end{array}$$

$\frac{12}{16} = \frac{3}{4}$

$\frac{11}{16} = .68\frac{3}{4}$ or $68\frac{3}{4}\%$.

Ans. $22 = 68\frac{3}{4}\%$ of 32.

17. What per cent of 17 is 5? 8? 11? 15? 21?

18. What per cent of 42 is 40? 41? 39? 2? 3?

19. What per cent of 125 is 123? 12? 118?

APPLIED PROBLEMS

1. A farmer raised 80 bushels of potatoes on an acre of land in 1915. In 1916 he raised 90 bushels. What was the increase? What was the per cent of increase?

SOLUTION.

$$\begin{array}{r} 90 \text{ (bu.) in 1916} \\ \underline{80} \text{ (bu.) in 1915} \\ 10 \text{ (bu.) increase} \end{array}$$

$\frac{10}{80}$ = what per cent (*i.e.* 10 = what per cent of 80)?

$\frac{10}{80} = \frac{1}{8} = 12\frac{1}{2}\%$.

Hence 10 bu. is $12\frac{1}{2}\%$ of 80 bu.

Ans. The increase was 10 bu.; the per cent of increase was $12\frac{1}{2}\%$.

2. John paid 50¢ for 100 papers. He sold them for $1.00. What was his gain? What was his gain per cent?

3. In a room of 40 pupils 12 are absent. What is the percentage of absence? Of attendance?

4. In a series of 5 games team A wins 4. What is the per cent of games won?

5. In a room of 42 pupils 28 are girls. What per cent of the pupils are girls?

6. In a school of 640 pupils 420 are boys. What per cent of the pupils are boys?

7. A clerk whose weekly salary is \$24 saves \$5. What per cent of his salary does he save?

8. A carload of wheat costs a grain dealer \$1750. He sells it for \$2000. What is his profit? What is his per cent of profit?

9. A fruit dealer pays \$3.00 for a bunch of bananas. He sells from this bunch $8\frac{1}{2}$ dozen at 30 cents a dozen; and 5 dozen at 25 cents a dozen. How much does he get for the bananas sold? What was his profit? What was his per cent of profit?

10. What is gained if a coat costing \$6.40 is sold for \$10? What per cent is gained?

11. A's salary in 1915 was \$1500. His salary in 1916 was \$1650. What was the increase? What was the per cent of increase?

12. The yield of wheat per acre in the United States in 1900 was 12 bushels; in 1910, 15.5 bushels. What was the increase? What was the per cent of increase?

13. The yield of potatoes in the United States per acre in 1900 was 93 bushels; in 1910, 106 bushels. What was the per cent of increase?

14. The yield of corn per acre in 1900 was 28 bushels; in 1910, 26 bushels. What was the decrease? What was the per cent of decrease?

15. The yield of oats per acre in 1900 was 32 bushels; in 1910, 28.6 bushels. What was the decrease? What was the per cent of decrease?

16. In September, flour was selling at \$9.25 per barrel; in November, flour was selling at \$12 per barrel. What was the per cent of increase?

17. In September, potatoes were selling at \$1.40 per bushel; in November, at \$2.00 per bushel. What was the per cent of increase?

18. In August, coal was retailing at \$8 per ton; in November, it was retailing at \$11.80 per ton. What was the per cent of increase?

19. The price of milk was raised 1 cent per quart on all grades October 1st. Before Oct. 1st

grade A was selling at 12¢,
grade B was selling at 10¢,
grade C was selling at 9¢.

What was the per cent of increase on each grade?

20. Of 300 grains of oats planted, 265 sprouted. What per cent sprouted?

21. Of 125 grains of wheat planted, 90 sprouted. What per cent sprouted?

22. Of 200 kernels of corn planted, 165 sprouted. What per cent sprouted?

23. What per cent of a mile is 100 yd.? 220 yd.? 440 yd.?

24. What per cent of a yard is 1 ft.? 2 ft. 1 in.? 2 ft. 6 in.?

25. What per cent of a gallon is 2 pints? 3 pints? 4 pints?

26. What per cent of a square yard is 144 sq. in.? 5 sq. ft.? 12 sq. ft.?

27. What per cent of a ton is 750 lb.? 1200 lb.? 1800 lb.?

28. A farmer set out 240 apple trees; 36 of them died. What per cent of the trees died?

29. A farmer set out 175 peach trees; 42 of them died. What per cent of the trees died?

30. A farmer set out 75 quince trees; 13 of them died. What per cent died?

31. A farmer set out 42 cherry trees; 8 of them died. What per cent died?

32. In 1915 a farmer paid his hired man \$300 for a year's work; in 1916 he paid him \$336 for a year's work. What was the per cent of increase?

33. The price of an automobile was reduced from \$550 to \$490. What was the per cent of reduction?

34. The same automobile was later reduced from \$490 to \$360. What was the per cent of reduction?

35. The price of an automobile was reduced from \$650 to \$595. What was the per cent of reduction?

36. The price of an automobile was increased from \$1280 to \$1350. What was the per cent of increase?

37. The price of an automobile was increased from \$1190 to \$1280. What was the per cent of increase?

38. Mr. Brown paid \$2000 for an automobile. Two years later he sold it for \$800; what was the per cent of loss?

39. The cost of an automobile was \$1950. After being used one year it was sold for \$1500. What was the per cent of loss?

40. The price of an automobile selling for \$1585 is raised \$175. What is the per cent of increase?

41. Of 5260 men injured in battle, 1370 died, 2420 recovered sufficiently to return, and the remainder were sent home. What per cent were there in each class?

50. Finding Per Cents of Numbers. The processes used are illustrated in the following exercises.

EXERCISES

1. On a spelling test of 20 words,

John missed 10%. How many words did he miss?

Mary missed 20%. How many words did she miss?

Henry missed 25%. How many words did he miss?

James missed 30%. How many words did he miss?

SPECIMEN SOLUTIONS. John missed 10% or $\frac{1}{10}$ of 20 words. $\frac{1}{10}$ of $20 = 2$

Ans. John missed 2 words.

Mary missed 20% or $\frac{1}{5}$ of 20 words.

$\frac{1}{5}$ of $20 = 4$

Ans. Mary missed 4 words.

2. On a spelling test of 50 words,

James spelled correctly 94%. How many words did he spell correctly?

Mary spelled correctly 90%. How many words did she spell correctly?

Henry spelled correctly 84%. How many words did he spell correctly?

John spelled correctly 80%. How many words did he spell correctly?

Jennie spelled 35 words correctly. What per cent did she spell correctly? What per cent did she misspell?

SPECIMEN SOLUTION. James spelled correctly 94% or .94 of 50 words.

$.94 \times 50 = 47.00$

Ans. James spelled correctly 47 words.

3. In a series of baseball games between four teams,

Team A won 75% of the games it played.
Team B won 60% of the games it played.
Team C won 50% of the games it played.
Team D won 15% of the games it played.

If 20 games were played by each team, how many games did each team win?

4. Find 75% of 5280 ft.
5. Find 90% of 320 rd.
6. Find $87\frac{1}{2}\%$ of 1728 cu. in.
7. Find $62\frac{1}{2}\%$ of 144 sq. in.
8. Find $66\frac{2}{3}\%$ of 1728 cu. in.
9. Find $66\frac{2}{3}\%$ of 231 cu. in.
10. Find $62\frac{1}{2}\%$ of 2000 lb.
11. Find $37\frac{1}{2}\%$ of 2150.42 cu. in.
12. Find 80% of 640 acres.
13. Find $66\frac{2}{3}\%$ of 135 cu. in.
14. Find 5% of \$874.20.
15. Find 6% of \$364.
16. Find $5\frac{1}{2}\%$ of \$152.50.
17. Find $112\frac{1}{2}\%$ of \$864.
18. Find 225% of \$1486.
19. Find $2\frac{1}{2}\%$ of \$3564.
20. Find $1\frac{1}{4}\%$ of 5280 ft.
21. Find 8.2% of 1728 cu. in.
22. Find 11.3% of 2150.42 cu. in.
23. Find 8.1% of \$756.42.
24. Find $\frac{1}{4}\%$ of 3,370,000.
25. Find $\frac{1}{8}\%$ of 670,000.

APPLIED PROBLEMS

1. A farmer had 375 sheep and sold 20% of them. How many did he sell?

2. In a school of 530 students 40% are boys. How many boys are there in the school?

3. A grain dealer bought 2480 barrels of flour and sold $12\frac{1}{2}\%$ of them. How many barrels did he sell?

4. A man's income is \$1200. He saves $16\frac{2}{3}\%$ of it. How much does he save?

5. A gasoline tank contained 260 gallons; 15% of it was sold. How many gallons were sold?

6. A furniture dealer offers to close out his entire stock at a reduction of 10% from the regular price. How much is the reduction on each of the following articles: table \$45? desk \$37.50? chair \$23.50? couch \$29.50? dining room set \$187?

7. An automobile cost \$1800. Since it was purchased it has decreased $16\frac{2}{3}\%$ in value. How much has it decreased in value? What is it worth now?

8. A farmer had 350 acres of land and sold 20% of it. How many acres did he sell? How many acres remained?

9. A man earns \$1500 in a year. He spends 90% of it. How many dollars does he spend? How much has he left?

10. In lumber mill No. (1) 34,497 board feet are sawed daily. Of this 5% are trimmed off. How many board feet are trimmed off?

11. In mill No. (3) 18,765 board feet are sawed daily; 9% are trimmed off. How many board feet are trimmed off?

12. An automobile selling for \$1395 is reduced 25%. Find the price after the reduction.

13. A's salary per year has been \$1780. He is given an increase in salary of 15%. What is his salary after the increase has been given him?

14. An automobile which costs \$1095 is offered for sale at a reduction of 15%. At what price is it offered?

15. In each of the following items, find how much the former price was reduced and what the per cent of reduction was on each article.

Article	Former Price	Selling Price	Amount Reduced	Reduction Per Cent
a. Mahogany sideboard	\$105.00	\$88.00	?	?
b. Mahogany china cabinet	110.00	89.00	?	?
c. Mahogany serving table	45.00	25.00	?	?
d. Mahogany sideboard	275.00	175.00	?	?
e. Mahogany china cabinet	178.00	98.00	?	?
f. Mahogany tea table	110.00	75.00	?	?
g. Mahogany china cabinet	350.00	265.00	?	?
h. Old oak sideboard	265.00	187.00	?	?
i. Old oak sideboard	175.00	162.50	?	?
j. Brown oak sofa	275.00	190.00	?	?
k. Brown oak sofa	200.00	185.00	?	?
l. Brown oak desk	325.00	195.00	?	?
m. Brown oak chair	60.00	30.00	?	?
n. Brown oak bookcase	225.00	158.00	?	?
o. Brown oak table	175.00	145.00	?	?
p. Brown oak table	225.00	155.00	?	?
q. Old oak table	275.00	175.00	?	?
r. Circassian walnut table	250.00	168.00	?	?
s. Mahogany card table	30.00	15.00	?	?
t. Mahogany secretary	150.00	98.00	?	?
u. Brown oak floor lamp	36.00	26.00	?	?
v. Mahogany bureau	250.00	125.00	?	?
w. Mahogany bed	200.00	98.00	?	?

II. Discount

51. A ***discount*** is a reduction made from a given price. This reduction is generally stated as so many per cent of the given price, which is called the *list price* or *marked price.*

The price after the discount has been taken off is called the *net price.*

APPLIED PROBLEMS

1. A teacher who orders books amounting to \$36.50 gets a discount of $16\frac{2}{3}\%$. What is the net price of these books?

2. A suit marked at \$22.50 is advertised at a discount of 10%. Find the net price.

3. An overcoat marked at \$42.50 is advertised at a discount of 10%. Find the net price.

4. The following are advertised at a reduction of $33\frac{1}{3}\%$ on these prices: \$3.50 satin; \$4.00 crepe; \$1.00 pongee; \$3.50 silk. Find the net price of each.

5. An ash can marked to sell at \$3.25 is reduced 12%. Find the net price.

6. A lawn mower marked to sell at \$10.50 is reduced $12\frac{1}{2}\%$. Find the net price.

7. A hammock marked to sell at \$15.60 is reduced 20%. Find the net price.

8. The marked price of a piano is \$425. A discount of 15% is allowed for cash. Find the net cost.

9. A farmer buys a reaper at \$85 less a discount of 5% for cash. What does he pay in cash for the reaper?

10. A victrola is sold for \$200, subject to a discount of $12\frac{1}{2}\%$ for cash. What is the net cost of the victrola if cash is paid?

11. A dozen copies of a book are ordered. The list price of the book is \$1.25 less 20%. Find the net cost of the 12 books.

12. A contractor orders 10 kegs of nails. There are 50 lb. of nails in each keg. The price per pound is $4\frac{1}{2}$ cents, less a discount of 15%. Find the net cost.

13. A farmer buys a corn harvester for \$135, less a discount of $12\frac{1}{2}\%$ for cash. How much money will he need if he pays cash for the harvester?

14. Oriental rugs are advertised for the August sale at discounts of 25%. Find the net price of each of the following: rug A listed at \$179; rug B listed at \$225; rug C listed at \$95; rug D listed at \$345.

15. An automobile listed at \$1595 is offered for quick sale at a discount of 15%. Find the net price offered.

16. Determine the amount of the discount and the net price on each article in the following list:

Article	Former Price	Per Cent of Discount	Discount	Net Price
a. Overcoat . .	\$20.00	15%	?	?
b. Overcoat . .	25.00	18%	?	?
c. Overcoat . .	30.00	$16\frac{2}{3}\%$	?	?
d. Overcoat . .	50.00	15%	?	?
e. Overcoat . .	65.00	$16\frac{2}{3}\%$	?	?
f. Sideboard . .	105.00	15%	?	?
g. Sideboard . .	275.00	40%	?	?
h. Sideboard . .	350.00	35%	?	?
i. Sideboard . .	265.00	30%	?	?
j. Bureau . .	250.00	48%	?	?
k. Bureau . .	165.00	52%	?	?
l. Table . . .	185.00	$12\frac{1}{2}\%$	?	?
m. Table . . .	205.00	54%	?	?
n. Table . . .	250.00	$37\frac{1}{2}\%$	?	?
o. Table . . .	320.00	35%	?	?

III. Interest

52. ***Interest*** is money paid for the use of money.

The ***principal*** is the sum of money loaned or borrowed.

The ***rate of interest*** is the per cent of the principal paid per year.

The ***amount*** is the sum of the principal and interest for the given time.

Example 1. A lends B \$500 at 5% for 1 year. How much does B owe A at the end of the year?

Solution. The *principal* is \$500. The *rate of interest* is 5%. The *interest* is 5% of \$500, or \$25. The *amount* due A at the end of the year is \$500 + \$25 = \$525.

Example 2. Mr. J. Smith borrows \$2500 from the bank for 60 days, interest at $5\frac{1}{2}$%. How much interest must he pay? (360 days are usually taken as the commercial year, and 30 days as the commercial month.)

Solution. 60 days = $\frac{60}{360}$ yr. = $\frac{1}{6}$ yr. The interest for one year is

$$\$2500 \times .05\tfrac{1}{2} = \$137.50.$$

Hence the interest for $\frac{1}{6}$ yr. is

$$\frac{1}{6} \text{ of } \$137.50 = \$22.92.$$

Summary: (1) *Multiply the principal by the given rate of interest.* (For example, $\$2500 \times .05\frac{1}{2} = \$137.50$.) *This product is the interest for one year.*

(2) *Express in days the time given. This number written over* 360 *gives the ratio of the time given to a year.* (For example, 60 days = $\frac{60}{360}$ yr.)

(3) *Multiply the interest found for one year by this fraction.* (For example, $\$137.50 \times \frac{1}{6} = \$22.92$.) *This product is the interest for the given time at the given rate.*

EXERCISES

Compute the interest on:

1. \$550 at 5% for 2 years.
2. \$4285 at 4% for 60 days.
3. \$650 for 6 mo. at 6%.
4. \$1098 for 7 mo. at 6%.
5. \$1476 at $4\frac{1}{2}$% for 9 mo.

NOTE. Where fractions appear cancellation may be used.

Thus, in example 5, the interest is

$$\frac{\overset{123}{\cancel{1476}}}{} \times \frac{9}{200} \times \frac{9}{\cancel{12}} = \frac{9963}{200} = \$49.82.$$

6. \$1764 at $5\frac{1}{2}$% for 8 mo.
7. \$2908 at $4\frac{1}{2}$% for 1 yr. 2 mo.
8. \$3052 at 5% for 3 mo. 15 da. ($\frac{105}{360}$ yr.).
9. \$2592 at $4\frac{1}{2}$% for 2 mo. 12 da.
10. \$2880 at 4% for 6 mo. 20 da.
11. \$3204 at $5\frac{1}{2}$% for 4 mo. 24 da.
12. \$4068 at 6% for 2 mo. 3 da.
13. \$6250 at 5% for 3 mo. 8 da.

Compute the interest and find the amount due on:

14. \$650 at 6% for 3 mo.
15. \$1275 at 5% for 5 mo.
16. \$1485 at 5% for 10 mo.
17. \$2250 at $5\frac{1}{2}$% for 8 mo. 8 da.
18. \$2750 at $5\frac{1}{2}$% for 6 mo. 12 da.
19. \$8788 at 4% for 3 mo.
20. \$85.60 at 4% for 4 mo.
21. \$3880 at $5\frac{1}{2}$% for 7 mo.
22. \$2960 at 5% for 5 mo.
23. \$3,000,000 at 4% for 4 mo. 12 da.

IV. Commission

53. ***Commission*** is the money paid to any one who transacts business for you. It is generally a percentage of the amount of money needed for a sale or purchase.

Thus a real estate agent who sells a house and lot for you may charge 5% of the selling price as his *commission* for doing the business.

If a commission is charged against the purchaser, the commission must be added to the price to obtain the total cost. If a commission is charged against the seller, it must be subtracted from the price to obtain the net price.

Example 1. A house and lot is sold for $5400 by a real estate agent at a commission of 5%. Find his commission. Find the net receipts of this sale.

Solution.

$$\begin{array}{r} \$5400 \\ .05 \\ \hline \$270.00 \end{array} \qquad \begin{array}{r} \$5400 \\ 270 \\ \hline \$5130 \end{array}$$

Ans. $270 commission; $5130 net receipts of sale.

Example 2. A traveling salesman sells for his firm goods to the amount of $16,500 in one year. His commission is $12\frac{1}{2}\%$ of his sales. Find his commission. Find the net receipts for these goods.

Solution.

$$12\tfrac{1}{2}\% = \frac{1}{8}$$

$$\frac{1}{8} \text{ of } \$16{,}500 = \$2{,}062.50$$

$$\begin{array}{r} \$16{,}500 \\ 2{,}062.50 \\ \hline \$14{,}437.50 \end{array}$$

Ans. $2,062.50 commission; $14,437.50 net receipts.

APPLIED PROBLEMS

1. An agent in New York sells a carload of hay for a farmer. The hay is sold for \$272.50 at a commission of 8%. Find the commission and the amount of money the agent should send the farmer.

2. An agent selling an automobile for \$595 receives a commission of 25%. What is his commission?

3. A house lot containing 8750 sq. ft. of land is sold by an agent at 23 cents per square foot. His commission is 2% of the selling price. Find his commission and the net receipts of the sale.

4. A book agent's commission is $37\frac{1}{2}\%$ of his sales. During the week he sells 28 books at \$2.50 each. How much is his commission and how much money should he send the company?

5. On an automobile selling at \$1095 an agent's commission is 20%. In one month he sells 12 cars. Find his commission.

6. A coal company gives a real estate agent a 20% commission on all orders for coal taken through his office. During the month he takes orders for 82 tons at \$8.95 per ton. Find his commission.

7. A clerk in a store receives a 10% commission on all orders given by people introduced by him. During a month such orders amount to \$325. What is his commission?

8. Mr. Jones is agent for victrolas. He receives a 20% commission on all his sales. He sells in one month 2 victrolas at \$25 each; 3 at \$50 each; 1 at \$175. What is his commission for the month?

9. An agent who sells farm machinery gets a commission of 15% of his sales. During the season he sells the following: 3 harrows at \$28.50 each; 5 mowers at \$38 each; 3 horse-rakes at \$26 each; 2 reapers at \$68 each. Find his commission.

10. A Farm Agency sells a farm for Mr. Brown for \$4300 at a commission of 8%. What are Mr. Brown's net receipts for the farm?

11. A dealer in Portland, Me., orders his agent in Chicago to buy him a carload of horses. The agent buys 28 horses at an average price of \$172.50 each. His rate of commission is 5%. What is his commission? What is the total cost of the carload of horses?

12. A grain dealer buys through his agent in Buffalo the following. Determine the agent's commission on each transaction and the gross cost in each case if the agent's rate of commission is $2\frac{1}{2}$%.

Article and Quantity	Price for Each	Commission	Gross Cost
a. 225 barrels B flour	\$8.75	?	?
b. 212 barrels A flour	9.10	?	?
c. 432 barrels C flour	8.50	?	?
d. 485 bushels wheat	1.73	?	?
e. 268 bushels corn	.82	?	?
f. 486 bushels corn	.85	?	?
g. 348 bushels oats	.46	?	?
h. 412 bushels corn	.43	?	?
i. 185 bushels clover seed	5.40	?	?
j. 226 bushels clover seed	5.56	?	?
k. 475 bushels timothy seed	2.85	?	?
l. 554 bushels timothy seed	2.93	?	?

CHAPTER V

MENSURATION

I. Areas

54. A ***square inch*** is a square having each side one inch long. A ***square foot*** is a square having each side one foot long. A ***square yard*** is a square having each side one yard long.

The surface enclosed by any figure may be expressed as a certain number of square inches or square feet or square yards, etc.

The square inch, the square foot, the square yard are known as the ***units of square measure***.

Table of Square Measure

144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards	= 1 square rod (sq. rd.)
160 square rods	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)

55. **Rectangles.** The process involved in finding areas of rectangles is illustrated by the following examples. The definition of a rectangle and the method of construction are given on p. 55.

Example 1. Draw a rectangle 3″ long and 2″ wide. Divide it by lines as in Fig. 29, p. 92.

The rectangle here is made up of 6 squares, each an inch long and an inch wide, or six *square inches*. There are two rows of three squares each.

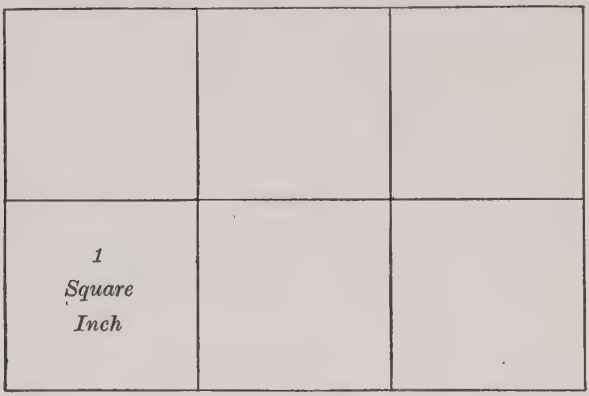

FIG. 29

The number of squares thus formed is called the ***area*** of the rectangle; for example, the area of the rectangle drawn above is 6 square inches.

If we let A stand for the number of square units in the area of a rectangle, b stand for the number of linear units in the base and h stand for the number of linear units in the height (or altitude), then

A (the number of square units) $= b \times h$ square units. When the multiplication of letters is indicated, the sign $\times$ is generally omitted.

The expression $A = bh$ is the ***formula for the area of a rectangle***. A *formula* is a brief way of expressing a rule.

EXAMPLE 2. The base of a rectangle is 4″ and its height is 3″. Find its area.

SOLUTION. Using the formula for the area of a rectangle, given the base and height in inches,

A(the number of square inches) = bh square inches.

Placing 4 for b and 3 for h,

$A = 4 \times 3$ square inches,

$A = 12$ square inches.

Ans. $A = 12$ sq. in.

EXAMPLE 3. The base of a rectangle is 7′ and its height is 6′. Find its area.

SOLUTION. A (the number of square feet) = bh square feet.

Placing 7 for b and 6 for h,

$A = 7 \times 6$ square feet,

$A = 42$ square feet.

Ans. $A = 42$ sq. ft.

EXAMPLE 4. The two sides of a rectangle are 1′ 6″ and 1′ 3″. Find its area.

SOLUTION. The base = $1\frac{1}{2}$′ and the height = $1\frac{1}{4}$′.

A (the number of square feet) = bh square feet.

$A = 1\frac{1}{2} \times 1\frac{1}{4}$ square feet,

(*Estimate:* 2×1 or 2 sq. ft.)

$A = \frac{3}{2} \times \frac{5}{4}$ square feet,

$A = \frac{15}{8}$ square feet,

$A = 1\frac{7}{8}$ square feet.

Ans. $A = 1\frac{7}{8}$ sq. ft.

EXAMPLE 5. The base of a rectangle is 1′ 2″ and the height is 8″. Find its area in square inches.

SOLUTION.

A (the number of square inches) = bh square inches.

$A = 14 \times 8$ square inches,

(*Estimate:* 15×8 or 120 sq. in.)

$A = 112$ square inches.

Ans. $A = 112$ sq. in.

EXAMPLE 6. The length of a rectangle is 3.4″ and the width is 2.5″. Find its area.

SOLUTION.

A (the number of square inches) $= bh$ square inches.

$A = 3.4 \times 2.5$ square inches $= 8.5$ square inches.

(*Estimate:* 3×3 or 9 sq. in.)

Ans. $A = 8.5$ sq. in.

EXERCISES

Copy and fill in the following table for areas of rectangles. Write the results for the last five decimally.

FORMULA: $A = bh$

	b	h	EST.	A
1.	5″	12″	60 sq. in.	60 sq. in.
2.	8′	5′		
3.	9″	6″		
4.	$4\frac{1}{2}$″	8″		
5.	$5\frac{1}{2}$′	4′		
6.	$10\frac{1}{2}$″	$5\frac{1}{2}$″	10×6 or 60 sq. in.	
7.	5.4″	6.5″	5×7 or 35 sq. in.	
8.	18.5′	12.4′	20×10 or 200 sq. ft.	
9.	.5″	1.8″		
10.	10″	3″		sq. ft.
11.	10″	1′ 2″		sq. in.
12.	2′ 6″	1′ 6″		sq. ft.
13.	1′ 2″	1′ 3″		sq. in.
14.	4.2′	2.7′		
15.	48.5′	112.2′	50×100 or 5000 sq. ft.	
16.	$3\frac{1}{2}$′	$2\frac{1}{4}$′		sq. ft.
17.	$8\frac{3}{4}$″	$6\frac{1}{2}$″		sq. in.
18.	9″	12″		sq. ft.
19.	15″	15″		sq. ft.
20.	$11\frac{3}{4}$″	$4\frac{1}{2}$″		sq. in.

21. A lot of land is 65′ by 30′. Draw to the scale of 20′ to 1″ a plan of the lot. Find the area of the lot.

22. A room is 13′ by 16′. Draw to the scale of 10′ to 1″ a plan of the room. Find the area of the room.

23. Draw to the scale of 100′ to the inch the plan of a lot of land 180′ by 125′. Find the area of the lot.

24. The four walls of a square room are each 3 yards high and 5 yards wide. How many square yards of paper will be required to paper the walls allowing for no waste? How many square yards would be required, if 10% extra were allowed for waste?

25. Each of the four panes of glass in a window measures 24″ by 15″. How many square inches of glass are there in the window? How many square feet?

26. The two blackboards in a certain class room are 3′ high. One board is 12′ long and the other is 15′ long.

(*a*) How many square feet of blackboard are there in the room?

(*b*) What is the ratio of the area of the smaller board to that of the larger?

(*c*) The smaller board is what per cent of the larger?

27. Measure each of the blackboards in your class room to the nearest tenth of a foot.

(*a*) How many square feet of board are there in your room?

(*b*) If there are two or more boards, what is the ratio of the smallest to the largest?

(*c*) What per cent is the smallest of the largest?

For exercises 27–30, the measurements should be made by several members of your class independently. These

measurements should be tabulated and averaged before the areas are computed. Make a table for each as follows:

	Length	Width
1.		
2.		
3.		
4.		
5.		
6.		
Average		

28. Measure the top of one desk to the nearest tenth of an inch.

(*a*) How many square inches does it contain?

(*b*) How many square feet does it contain?

(*c*) How many square feet of wood are there in the tops of all the pupils' desks in the room?

29. Measure the floor of the class room to the nearest tenth of a foot.

(*a*) How many square feet does it contain?

(*b*) Compare the blackboard space with the floor space; that is, find the ratio.

30. Measure one window pane to the nearest tenth of an inch.

(*a*) Find the area of one pane of glass.

(*b*) Find the total area of all the panes.

(*c*) Compare it with the floor space.

(*d*) The window area is what per cent of the floor area?

31. One room is 18′ long and 15′ wide. A second room is 12′ long and 15′ wide.

(*a*) Find the areas of the floors.

(*b*) The area of the floor of the second room is what fractional part of the area of the floor of the first? What per cent?

(*c*) The first area is how many square feet larger than the second?

(*d*) The difference obtained in (*c*) is what fractional part of the floor area of the first room? What per cent?

(*e*) The difference obtained in (*c*) is what fractional part of the floor area of the second room? What per cent?

32. A certain class room is 40′ by 30′. The assembly hall is 120′ by 80′.

(*a*) Find the floor area of each.

(*b*) The floor area of the class room is what fractional part of the floor area of the assembly hall? What per cent?

(*c*) What is the difference in the areas?

(*d*) The difference obtained in (*c*) is what fractional part of the floor area of the assembly hall? What per cent?

(*e*) The difference obtained in (*c*) is what fractional part of the floor area of the class room? What per cent?

33. On a map drawn to the scale of 200 miles to the inch, what area would be inclosed by a strip 3″ by 2″?

34. On a map drawn to the scale of 50 miles to the inch, what area would be inclosed by a strip $1\frac{1}{2}$″ by 3″?

35. On a map drawn to the scale of 40 miles to the inch, what area would be inclosed by a strip 1.4″ by 2.2″?

36. A lawn 36′ by 30′ is to be sodded. How many pieces of sod will be needed, if each piece is 12″ by 15″?

37. Find the cost of a cement sidewalk 40′ long and 4′ 9″ wide at 22 cents a square foot.

38. Find the cost of a cement sidewalk 60′ long and 4′ 6″ wide at 20 cents a square foot.

39. A rectangular lot of land 60′ by 50′ has a walk 5′ wide around it.

(*a*) What is the area of the lot?

(*b*) What is the area of the walk?

(*c*) The area of the walk is what per cent of the area of the lot?

(*d*) Find the cost of cementing the walk at 20 cents a square foot.

40. The area of a rectangle is 6 sq. in., its base is 3″ long. Find its height.

SOLUTION. Place 6 for A and 3 for b in the formula,

$$A = bh$$
$$6 = 3h \qquad (\textit{Estimate} = 2'')$$

Dividing each member by 3,

$$2 = h$$

CHECK. $6 = 3 \times 2$

Ans. $h = 2''$.

The rectangle in Fig. 29, p. 92, contains 6 square inches. The base is 3 inches long, so the rectangle contains 3 columns of squares having 2 squares in each column, hence the height is 2″.

41. The area of a rectangle is $12\frac{1}{2}$ sq. in.; the height is $2\frac{1}{2}''$. Find its base.

SOLUTION. Place $12\frac{1}{2}$ for A and $2\frac{1}{2}$ for h in the formula,

$$A = bh$$
$$12\tfrac{1}{2} = b \times 2\tfrac{1}{2} \qquad (\textit{Estimate} = 5'')$$

Dividing each member by $2\frac{1}{2}$,

$$5 = h$$

CHECK. $12\frac{1}{2} = 2\frac{1}{2} \times 5$

Ans. $b = 5''$.

Following the models in examples 40 and 41, find the required values in the following table. Copy the table and fill in the values.

FORMULA: $A = bh$

	A		EST.	EQUATION	?
42.	50 sq. in.	$b = 10''$	$5''$	$50 = 10h$	$h = 5''$
43.	90 sq. ft.	$h = 7\frac{1}{2}'$		$90 = 7\frac{1}{2}b$	$b = 12'$
44.	150 sq. in.	$b = 15''$			$h = ?$
45.	6.5 sq. in.	$h = 5''$			$b = ?$
46.	$12\frac{1}{2}$ sq. ft.	$b = 2\frac{1}{2}'$			$h = ?$
47.	225 sq. in.	$b = 1'\ 3''$			$h = ?$
48.	18.9 sq. in.	$h = 3.15''$			$b = ?$
49.	$18\frac{3}{8}$ sq. ft.	$h = 3\frac{1}{2}'$			$b = ?$

50. Find the length of a gymnasium floor 45 feet wide that contains 4050 sq. ft.

51. A football field contains 52,800 sq. ft., and is 330′ long. How wide is it?

52. A lot of land contains 2394 sq. ft., and its length is 36′. How wide is it? How long must the fence be that incloses it?

53. Out of a piece of cardboard, 24″ by 36″, small cardboards are to be cut 18″ long and having a surface measure of 216 sq. in. How many of these smaller boards can be cut out of each large board?

54. For a room 12′ by 16′ find which width of printed linoleum would be most economical, if the choice is to be made from the following:

(*a*) 6′ wide at \$1.60 per running yard,

(*b*) 9′ wide at \$2.30 per running yard,

(*c*) 12′ wide at \$3.00 per running yard.

Find the total cost in each case.

56. Squares. A ***square*** is a special kind of rectangle. The base and height of the square are the same. If s denotes the number of linear units in the base, then s can also denote the number of linear units in the height, hence the formula for the area of the square becomes:

$$A = s \times s.$$

This is commonly written

$$A = s^2 \text{ (read "the square of } s\text{")}.$$

EXAMPLE 1. Find the area of a square if each side is 8″.

SOLUTION. A (the number of square inches) $= s^2$ square inches.

Placing 8 for s,

$A = 8^2$ square inches,
$A = 8 \times 8$ square inches,
$A = 64$ square inches.

Ans. $A = 64$ sq. in.

EXAMPLE 2. Find the area of a square if each side is 5.4′.

SOLUTION. A (the number of square feet) $= s^2$ square feet.

$A = 5.4^2$ square feet, (*Estimate*: 5×6 or 30 sq. ft.)
$A = 5.4 \times 5.4$ square feet,
$A = 27.16$ square feet.

Ans. $A = 27.16$ sq. ft.

EXERCISES

1. Find the area of a square if each side is 12″.

2. Find the area of a square in square feet if each side is 1′ 6″.

3. Find the area of a square in square inches if each side = 1′ 2″. How many square feet in this area?

4. Copy and fill in the following table for areas of squares.

FORMULA: $A = s^2$

s	EST.	A
a. 7″		
b. 5.3″	5×6 or 30 sq. in.	
c. $8\frac{1}{2}$″		
d. $6\frac{1}{4}$″		
e. 1′ 4″		sq. in.
f. 3′ 3″		sq. ft.
g. 12′ 6″		sq. ft.
h. 2 yd. 9 in.		sq. yd.
i. 150 rd.		acres
j. 48.2′		sq. ft.

Express answers to (*f*), (*g*), (*h*), (*i*), (*j*) decimally.

5. The side of one square is $7\frac{1}{2}$″; the side of another square is $12\frac{1}{2}$″.

(*a*) Find their perimeters.

(*b*) Find the ratio of their perimeters.

(*c*) The first perimeter is what per cent of the second?

(*d*) Find their areas.

(*e*) Find the ratio of their areas.

(*f*) The first area is what per cent of the second?

(*g*) The second is how many square inches larger than the first?

(*h*) The second is what per cent larger than the first?

6. A square lot of land is 84′ long on each side. Draw a plan of the lot tc the scale of 20′ to the inch. Find the area of the lot.

7. A square lot of land is 20 rods long on each side. How many square rods are there in its area? How many acres?

8. A man owns two lots of land, one of which is 40′ by 40′ and the other is 25′ by 64′.

(*a*) Find their areas.

(*b*) Find the ratio of their areas.

(*c*) Which would require the longer fence to inclose it? How much longer?

9. One lot of land is found to be 54′ by 24′. A second lot of land is found to be 36′ by 36′.

(*a*) Find the ratio of their areas.

(*b*) Which would require the longer fence to inclose it? How much longer?

(*c*) The area of the square is what per cent of the area of the rectangle?

(*d*) The perimeter of the square is what per cent of the perimeter of the rectangle?

10. Find the side of a square whose area is 25 sq. in.

SOLUTION. Place 25 for A in the formula,

$$A = s^2,$$

then, $25 = s^2$

Taking the square root, $5 = s$

Since s^2 means $s \times s$, you find one of the two equal factors of 25; that factor, 5, is called the ***square root*** of 25.

CHECK. $25 = 5^2$ or 5×5

Ans. $s = 5''$.

11. Find the side of a square whose area is 144 sq. in.

SOLUTION. Place 144 for A in the formula,

$$A = s^2,$$

then, $144 = s^2$

Taking the square root, $12 = s$

CHECK. $144 = 12^2$ or 12×12

Ans. $s = 12''$.

12. Following the model in examples 10 and 11, find the required values in the following table. Copy the table and fill in the values.

FORMULA: $A = s^2$

	s	EST.	A
a.	4″	4″	16 sq. in.
b.			36 sq. ft.
c.			81 sq. in.
d.			49 sq. yd.
e.			64 sq. in.
f.			121 sq. in.
g.			400 sq. yd.
h.			1,600 sq. ft.
i.			196 sq. in.
j.			225 sq. ft.
k.			4,900 sq. ft.
l.			90,000 sq. ft.
m.			256 sq. rd.
n.			324 sq. ft.

13. The area of a square field is 100 sq. rd. How long is it? How long a fence will be required to inclose it?

14. A square picture contains 36 sq. in. What are the inside dimensions of the frame that contains the picture?

15. A trunk has a square end and the area of the end is 576 sq. in. How long must the shortest strap be that will go around the trunk?

16. The area of a square is 225 sq. in.

(*a*) How long is its perimeter?

(*b*) A rectangle is 20″ long and has the same perimeter as the square. What is its width?

(*c*) Find the area of the rectangle.

(*d*) The area of the rectangle is how much smaller than the area of the square?

(*e*) The area of the rectangle is what per cent smaller?

57. Parallelograms. Figure 30 is a parallelogram having b for its base and h for its height (or altitude).

The height h divides the parallelogram into two parts marked I and II. Part I is a triangle. If triangle I is

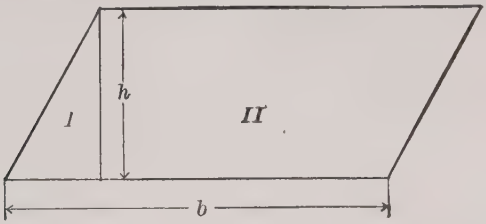

FIG. 30

taken from the position in Fig. 30 and placed in the position shown in Fig. 31, a rectangle is formed having the same base and altitude as the parallelogram in Fig. 31. The

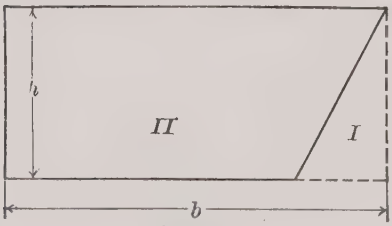

FIG. 31

two figures have the same areas, for they are made up of the same two parts I and II.

For the rectangle, we have

A (the number of square units) = bh square units.

Hence, for the parallelogram,

A (the number of square units) = bh square units.

EXERCISES

1. Find the area of the parallelogram having its base $4\frac{1}{2}''$, and its height $5''$.

2. Find the area of a parallelogram having its base $7\frac{1}{2}''$ and its height $5\frac{1}{2}''$.

3. Two sides of a parallelogram are $2.3''$ and $1.8''$; they include an angle of $50°$. Draw the parallelogram. Draw the altitude.

(*a*) Measure the height to the nearest tenth of an inch.

(*b*) Find the area of the parallelogram.

4. Two sides of a parallelogram are $2\frac{1}{2}''$ and $2.1''$; they include an angle of $40°$. Draw the parallelogram. Draw the altitude.

(*a*) Measure the height to the nearest tenth of an inch.

(*b*) Find the perimeter of the parallelogram.

(*c*) Find its area.

5. Two sides of a parallelogram are $2''$ and $1.8''$; they include an angle of $35°$. Draw the parallelogram. Draw the altitude.

(*a*) Measure the height to the nearest tenth of an inch.

(*b*) Find the area of the parallelogram.

(*c*) Find the area of the rectangle having the same base and altitude as this parallelogram.

(*d*) If $2''$ and $1.8''$ were the sides of a rectangle, what would its area be?

6. Two sides of a parallelogram are $3.2''$ and $4.1''$; they include an angle of $120°$. Draw the parallelogram Draw the altitude.

Measure the height. Find the perimeter and the area of the parallelogram.

58. Triangles. Figure 32 is a triangle having b for its base and h for its height (or altitude).

Figure 33 is a parallelogram having the same base and altitude as the triangle in Fig. 32. The dotted line

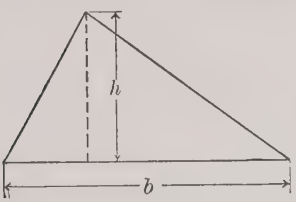

FIG. 32

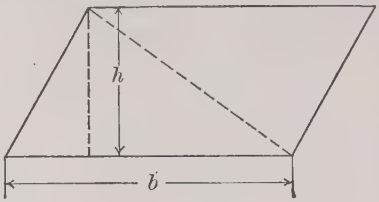

FIG. 33

divides the parallelogram into two triangles. If you make a tracing of the triangle in Fig. 32 and place it upon each of the triangles formed by the dotted line in Fig. 33, you will find that it exactly fits upon each one. Thus the triangle has an area equal to one half of the area of the parallelogram.

For the parallelogram,

A (the number of square units) = bh square units.

Hence, for the triangle,

A (the number of square units) = $\frac{1}{2}\ bh$ square units.

In this formula $A = \frac{1}{2}\ bh$ (read $\frac{1}{2}$ the product of b and h), note that there are three distinct factors in the second member, $\frac{1}{2}$ and b and h.

EXAMPLE 1. Find the area of a triangle whose base is 8″ and height 6″.

SOLUTION. A (the number of square inches) $= \frac{1}{2}\ bh$ square inches.

Placing 8 for b and 6 for h,

$A = \frac{1}{2} \times 8 \times 6$ square inches,

$A = 24$ square inches. *Ans.* $A = 24$ sq. in.

NOTE. You will see that it makes no difference in the answer whether you multiply 8 by 6 and take one half the product, or multiply 8 by one half of 6.

EXAMPLE 2. The base of a triangle is $13\frac{1}{2}'$, and the height is $10.4'$; find its area.

SOLUTION. A (the number of square feet) $= \frac{1}{2}\,bh$ square feet.

$A = \frac{1}{2} \times 13.5 \times 10.4$ square feet,

(*Estimate:* 7×10 or 70 sq. ft.)

$A = 70.2$ square feet. *Ans.* $A = 70.2$ sq. ft.

[Here it is somewhat easier to multiply 13.5 by one half of 10.4.]

EXERCISES

Copy and fill in the following table for areas of triangles.

FORMULA: $A = \frac{1}{2}\,bh$

	b	h	EST.	A
1.	$8''$	$7''$		
2.	$3'\ 6''$	$4'$		
3.	$2'\ 3''$	$2'\ 6''$		
4.	$5\frac{1}{2}''$	$6\frac{1}{2}''$	3×6 or 18 sq. in.	
5.	$3'\ 6''$	$4'\ 6''$		sq. ft.
6.	$3.7''$	$2.4''$		
7.	$13.5'$	$8.6'$		
8.	$7\frac{1}{4}''$	$12''$		
9.	$1'\ 1''$	$9''$		sq. in.
10.	$4\frac{3}{4}''$	$2\frac{1}{4}''$		

11. Find the area of a triangle if its base is $8\frac{3}{4}''$ and its height $4\frac{1}{2}''$.

12. Find the number of square feet in the gable end of a house in the form of a triangle whose base is $32'$ and height $12'$.

13. (*a*) Find the area of the lot of land represented by the triangle in Fig. 34.

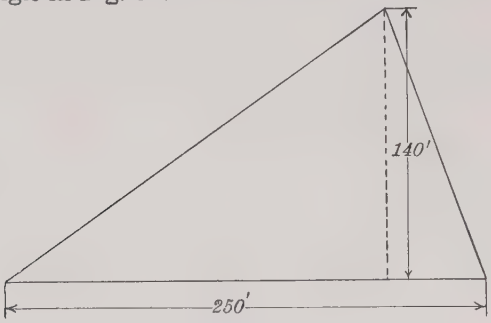

Fig. 34

(*b*) By measuring the base and height find the scale used in the drawing.

14. Figure 35 is the plan of a field drawn to a certain scale.

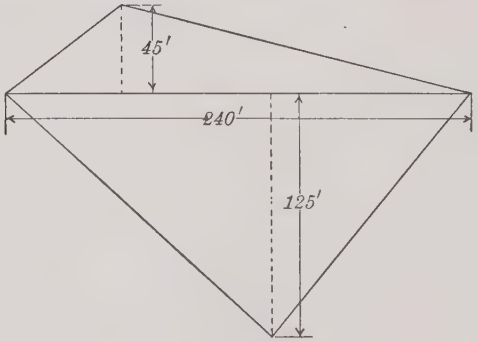

Fig. 35

(*a*) Find the scale used.
(*b*) Find the area of the larger triangle.
(*c*) Find the area of the smaller triangle.
(*d*) How many square feet are there in the field?

15. Figure 36 is the plan of a field. Find the area.

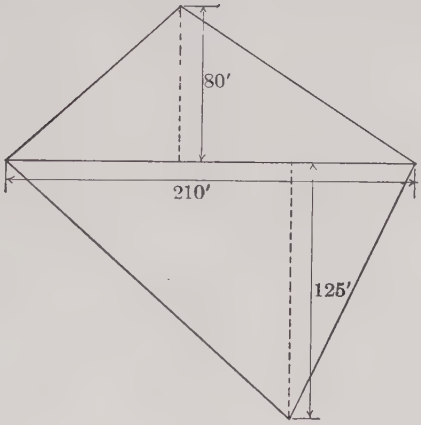

FIG. 36

16. The area of a triangle is 36 sq. ft. The base is 18′; find its height.

SOLUTION. Placing 36 for A and 18 for b in the formula,

$$A = \tfrac{1}{2}\,bh,$$

then, $36 = \tfrac{1}{2} \times 18\,h$ (*Estimate* = 4′)

$36 = 9\,h$ CHECK. $36 = 9 \times 4$

Dividing each member by 9,

$4 = h$ *Ans.* $h = 4'$.

17. The area of a triangle is 120 sq. in., and its height is 18″. Find its base.

SOLUTION. Placing 120 for A and 18 for h in the formula,

$$A = \tfrac{1}{2}\,bh,$$

then, $120 = \tfrac{1}{2}\,b \times 18$ (*Estimate* = 13″)

Dividing each member by 9,

$13\tfrac{1}{3} = b$ *Ans.* $b = 13\tfrac{1}{3}''$.

18. Find the height of a triangle if its area is 40 sq. ft., and its base is 20′.

19. Find the base of a triangle if its area is 35 sq. in., and its height is 14″.

20. Find the base of a triangle if its area is $42\frac{1}{2}$ sq. ft., and its height is 15′.

21. The area of a triangle = $15\frac{1}{2}$ sq. in., and its height is 20″. How long is its base?

22. The area of a triangle is $3\frac{3}{4}$ sq. in., and its height is 5″. How long is its base?

23. The base of a triangle is 10″. It has the same area as a square whose side is 6″. Find the height of the triangle.

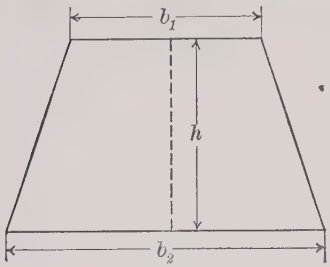

FIG. 37

59. Trapezoids. A ***trapezoid*** is a plane figure inclosed by four lines only two of which are parallel.

In the trapezoid of Fig. 37, the parallel sides, b_1 (read b, sub one) and b_2 (read b, sub two), are the *bases;* h, which is perpendicular to the bases, is the *height* (or *altitude*).

EXERCISES

1. Measure the sides and altitude of the trapezoid in Fig. 37 to the nearest *hundredth* of an inch.

(*a*) What is the sum of the two bases?

(*b*) What is the perimeter?

(*c*) Measure the four angles with your protractor. What is the sum of the four angles?

(*d*) What is the ratio of the upper base to the lower base?

(*e*) b_2 is what per cent longer than b_1? (Answer to the nearest per cent.)

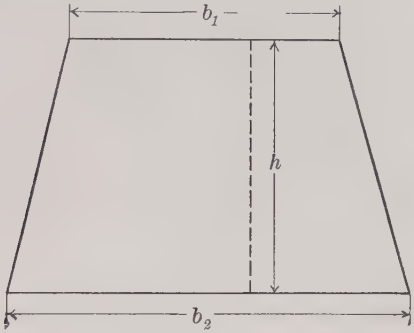

FIG. 38

2. Measure the sides and altitude of trapezoid in Fig. 38 to the nearest hundredth of an inch. Find all results required in Ex. 1.

60. Area of Trapezoid. Part I of Fig. 39 is a trapezoid having b_1 and b_2 for its bases and h for its altitude. If

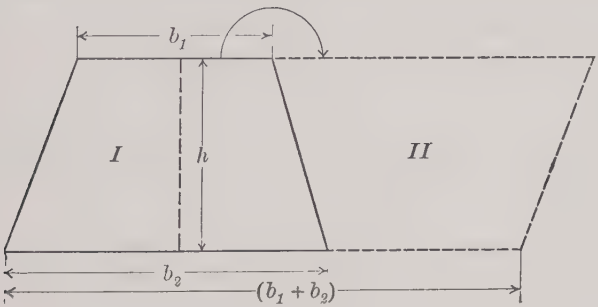

FIG. 39

trapezoid I is turned so that b_2 lies on b_1 extended, then a parallelogram is formed having $(b_1 + b_2)$ for its base and

h for its altitude. Note that trapezoid I takes the position of trapezoid II.

For the parallelogram,

A (the number of square units) $= (b_1 + b_2)\, h$ square units.

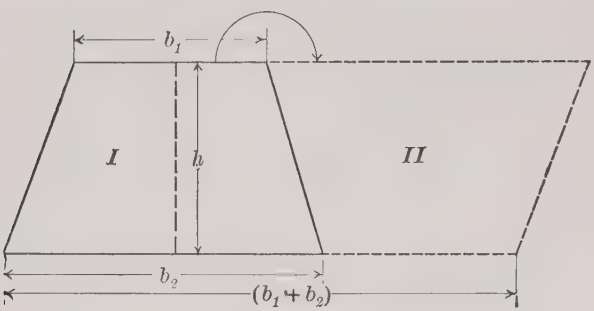

FIG. 39

Hence, for the trapezoid, which is one half of the parallelogram,

A (the number of square units) $= \frac{1}{2}(b_1 + b_2)\, h$ square units.

The formula $A = \frac{1}{2}(b_1 + b_2)\, h$ should be read, A equals one half the sum of b_1 and b_2 times h.

Note that there are three distinct factors: $\frac{1}{2}$, and $(b_1 + b_2)$, and h.

The signs of grouping ( ) are called ***parentheses.***

EXAMPLE 1. Find the area of a trapezoid whose bases are $15''$ and $9''$ and whose height is $4\frac{1}{2}''$.

SOLUTION. A (the number of square inches) $= \frac{1}{2}(b_1 + b_2)h$ square inches.

Placing 15 for b_1, 9 for b_2, and $4\frac{1}{2}$ for h,

$A = \frac{1}{2}(15 + 9) \times 4\frac{1}{2}$ sq. in., (*Estimate* = 50 sq. in.)

$A = \frac{1}{2} \times 24 \times \frac{9}{2}$ sq. in.,

$A = 54$ sq. in. ***Ans.*** $A = 54$ sq. in.

EXAMPLE 2. The bases of a trapezoid are $3\frac{1}{2}''$ and $4\frac{1}{2}''$. The height is $5\frac{1}{4}''$. Find its area.

SOLUTION. A (the number of square inches) $= \frac{1}{2}(b_1 + b_2)h$ square inches.

Placing $3\frac{1}{2}$ for b_1, $4\frac{1}{2}$ for b_2, and $5\frac{1}{4}$ for h,

$A = \frac{1}{2}(3\frac{1}{2} + 4\frac{1}{2}) \times 5\frac{1}{4}$ sq. in., (*Estimate* = 20 sq. in.)

$A = \frac{1}{2} \times 8 \times \frac{21}{4}$ sq. in.,

$A = 21$ sq. in. *Ans.* $A = 21$ sq. in.

EXERCISES

Copy and fill in the following table for areas of trapezoids.

FORMULA: $A = \frac{1}{2}(b_1 + b_2)h$

	b_1	b_2	h	EST.	A
1.	8″	6″	5″	35 sq. in.	35 sq. in.
2.	13″	9″	$4\frac{1}{2}''$	50 sq. in.	
3.	$15\frac{1}{2}''$	$12\frac{1}{2}''$	$5\frac{1}{2}''$		
4.	5.4″	3.5″	2.8″		
5.	142′	133′	92′		
6.	16.4′	32.5′	20.7′		
7.	1′ 6″	1′ 3″	2′		
8.	8.7″	4.9″	6.7″		

9. Find the area of a lot of land the shape of a trapezoid, if the parallel sides are 240′ and 80′ and the altitude is 125′. How many acres are there in the lot?

10. Find the area of the floor of which this diagram (Fig. 40) is a plan. (The scale is 15′ to the inch.)

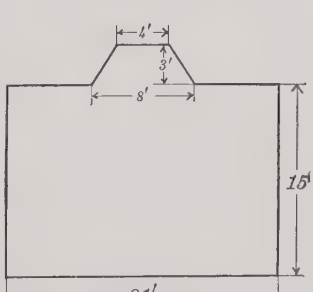

FIG. 40

II. Representation of Number Data by Rectangular Graphs

61. The following examples illustrate the use of rectangular graphs to represent quantities.

Example 1. The number of immigrants to the United States in 1912, 1913, 1914, and 1915 is represented by the following rectangular graphs, 1″ of length representing 500,000 people.

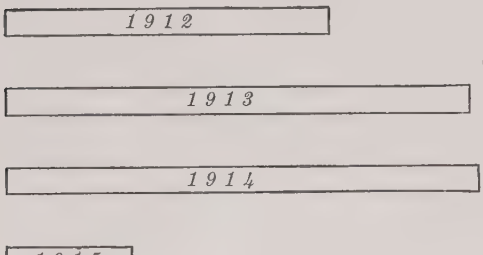

Fig. 41

1912: 1.68″
$1.68 \times 500{,}000 =$ 840,000

1913: 2.40″
$2.40 \times 500{,}000 =$ 1,200,000

1914: 2.44″
$2.44 \times 500{,}000 =$ 1,220,000

1915: 0.65″
$0.65 \times 500{,}000 =$ 325,000

Hence the number of immigrants in these years was as follows:

1912:	840,000 immigrants.
1913:	1,200,000 immigrants.
1914:	1,220,000 immigrants.
1915:	325,000 immigrants.

These numbers are *round numbers*, or *approximate numbers*. See page 64.

Example 2. The populations of the eight largest cities in the world (Census 1910) are represented by the following rectangular graphs, 1″ of length representing 2,000,000

people. Measure the lengths of the rectangles and find the population of each city in round numbers.

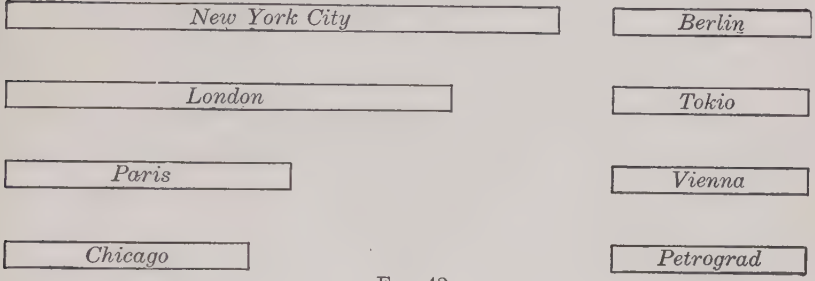

FIG. 42

EXAMPLE 3. The total exports from the United States in 1914 were \$2,364,579,148 and in 1915 were \$2,768,589,340.

The total imports in 1914 were \$1,893,925,657 and in 1915 were \$1,674,169,740. Represent each of these by rectangular graphs, letting 1″ of length represent \$1,000,000,000.

Exports in 1914:

$$\frac{2{,}364{,}579{,}148}{1{,}000{,}000{,}000} = 2.36 \text{ to nearest hundredth}$$

Exports in 1915:

$$\frac{2{,}768{,}589{,}340}{1{,}000{,}000{,}000} = 2.77 \text{ to nearest hundredth}$$

Imports in 1914:

$$\frac{1{,}893{,}925{,}657}{1{,}000{,}000{,}000} = 1.89 \text{ to nearest hundredth}$$

Imports in 1915:

$$\frac{1{,}674{,}169{,}740}{1{,}000{,}000{,}000} = 1.67 \text{ to nearest hundredth}$$

From these results we may construct a rectangular graph on the given scale, so that 1″ length represents $1,000,000,000. The resulting figure is shown below.

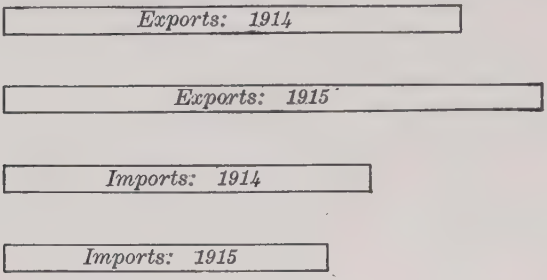

FIG. 43

EXERCISES

1. In the public high schools of the United States in 1914 there were 541,486 boys and 677,318 girls. Represent these by rectangular graphs, letting 1″ of length represent 200,000.

2. The number of pupils enrolled in the schools of the United States in 1914 was 19,153,786 and the average daily attendance was 14,216,459. Represent these by rectangular graphs, letting 1″ of length represent 1,000,000.

3. By rectangular graphs compare the estimated populations of the following cities in the United States (1915):

New York	5,468,190
Philadelphia	1,683,664
Chicago	2,447,045
St. Louis	745,988
Boston	745,139
Cleveland	656,975

Let 1″ represent 1,000,000.

III. Circles

62. A ***curved line*** is a line no part of which is straight.

A ***circle*** is a curved line all points of which are equally distant from a point within called the ***center***. Any part of the curve is an ***arc***. The ***radius*** of a circle is a straight line drawn from the center to any point in the curve. The ***diameter*** is a straight line joining two points of the curve and passing through the center.

Sometimes the surface inclosed by the curved line is called the circle, and the line itself is called the *circumference*.

The ***circumference*** of the circle is the length of the circle.

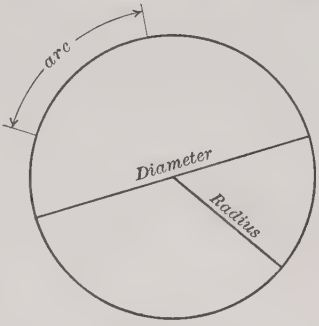

Fig. 44

How are the radii (plural of radius) of a circle related to each other? What is the ratio of the diameter of a circle to its radius?

Using a tape, find the distance around a tomato can (or any other object of the same shape). Measure the diameter of the bottom of the can. Divide the number of inches in the circumference by the number of inches in the diameter. What is the ratio?

You find that the circumference of the bottom is a little more than 3 times as long as the diameter. If you were able to find how many times the circumference contained the diameter more accurately, you would find it to be 3.1416, or nearly $3\frac{1}{7}$ times. This mixed decimal 3.1416

is commonly expressed by the symbol π (read *pi*). Hence to find the circumference of a circle multiply the diameter by π.

EXERCISES

1. Write the formula for the circumference of a circle when the diameter is given. Explain each letter in the formula.

2. Write the formula for the circumference of a circle when the radius is given.

63. Area of a Circle. The area of the circle is the number of square units of surface inclosed by the circle.

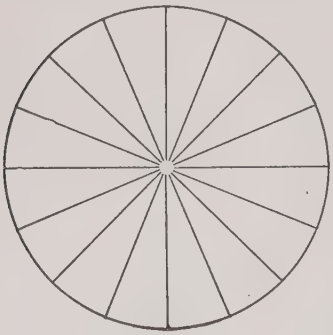

FIG. 45

Cut the surface inclosed by a circle into any number of equal parts, say 16, as shown in Fig. 45. Fit them together as shown in Fig. 46. Figure 46 resembles a parallelogram, having for its base one half the circumference of a circle ($\frac{1}{2}$ c), and for its height the radius of the circle.

For the parallelogram, we have

A (the number of square units) = bh square units.

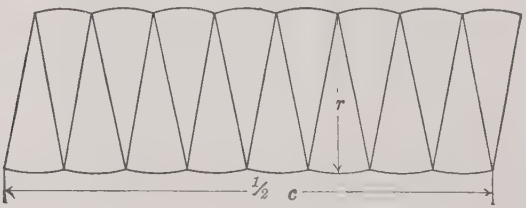

FIG. 46

Hence, for the circle,

A (the number of square units) $= \frac{1}{2}\ cr$ square units,

where c = the circumference of the circle (in linear units),

and r = the radius (in linear units).

But $c = 2\pi r$,

whence, $A = \frac{1}{2} \times 2\pi r \times r$,

or $A = \pi r^2$,

where $\pi = 3\frac{1}{7}$.

In Fig. 47 there are four squares formed having r for each side, hence their combined area will be $4r^2$. It will

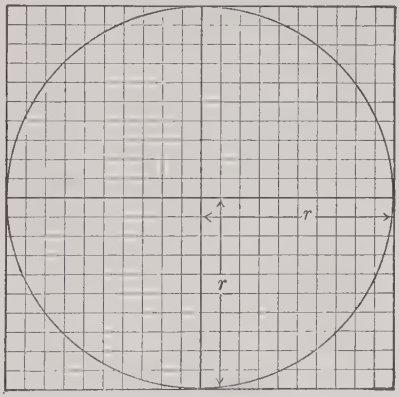

Fig. 47

be seen that the area of the circle is somewhat less than $4r^2$. The circle drawn has a radius of one inch. Counting the small squares (hundredths of a square inch) in one quarter of the circle, there are seen to be between 75 and 80. Multiplying this by 4, you get between 3 and 3.2 sq. in. for the area of the circle.

Example 1. The radius of a circle is $2\frac{1}{4}''$. Find the circumference and area of the circle to three figures.

(Use 3.14 for π.)

SOLUTION.

$c = 2\pi r$

$c = 2 \times 3.14 \times 2.25''$ (*Estimate* = 14″)

$c = 14.13''$

Ans. $c = 14.1''$.

$A = \pi r^2$

$A = 3.14 \times (2\frac{1}{4})^2$ sq. in. (*Estimate* = 16 sq. in.)

$A = 15.89$ sq. in.

Ans. $A = 15.9$ sq. in.

EXERCISES

Copy the following table and fill in the required values to three figures.

	d	r	EST. c	$c = 2\pi r$	EST. A	$A = \pi r^2$
1.	6.2″	3.1″	19″	19.5″	30 sq. in.	30.2 sq. in.
2.		6.3″				
3.		4.4″				
4.	12.8′					
5.		4.9″				
6.		31.3′				
7.	$7\frac{3}{4}''$					
8.		$1\frac{3}{8}''$				
9.	$12\frac{1}{4}''$					

10. The circumference of a circle is $11\frac{1}{2}''$ long. Find its radius to three figures.

SOLUTION. Placing 11.5 for c and 3.14 for π in the formula,

$c = 2\pi r$

$11.5 = 2 \times 3.14\, r$ (*Estimate* = $1\frac{3}{4}''$)

Dividing each member by 6.28,

$1.831 = r$ *Ans.* $r = 1.83''$.

11. The circumference of a circle is 47.4′. Find its diameter to three figures.

12. The circumference of a circle is 134′. Find its radius to three figures.

13. The diameter of a circular grass plot is 42′.

(*a*) Find the number of feet around it.

(*b*) Find the number of square feet of turf in it.

(*c*) A 3′ walk surrounds it. Find the number of square feet covered by the grass and walk.

(*d*) Find the area of the walk.

(*e*) The area of the walk is what per cent of the total area?

14. A circular table top has a diameter of 30″. A square table top has sides 30″ long.

(*a*) Find the perimeter of the top of each table.

(*b*) Find what ratio the perimeter of the circular top has to the perimeter of the square top.

(*c*) Find what per cent the area of the circular top is of the area of the square top.

IV. Representation of Number Data by Circular Graphs

64. The unit of measure for a circle is an arc of one *degree*. The circle contains 360 of these degree-arcs. To represent number data by circular graphs you must find out what part of a circle, or how many degrees, each number represents.

A circular graph can only be made when you have the total of the groups to be represented.

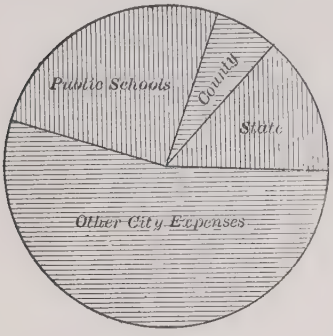

Fig. 48

Example 1. The tax rate for a certain city for 1916 was \$17.80 per \$1000 valuation. This circular graph represents how it was divided. Find the approximate amount per \$1000 valuation for each purpose named.

With the protractor find the number of degrees in each angle. The number of degrees in the arc will be the same.

State tax:	50°, $\frac{50}{360}$ of 17.80 =	\$2.47
County tax:	24°, $\frac{24}{360}$ of 17.80 =	\$1.19
Public schools:	90°, $\frac{90}{360}$ of 17.80 =	\$4.45
Other city expenses:	196°, $\frac{196}{360}$ of 17.80 =	\$9.69
Circle:	360°, total tax:	\$17.80

Example 2. Represent by a circular graph the following data from the United States Census, 1910:

Native born whites of native parentage . .	49,488,575
Native born whites of foreign parentage . .	18,897,837
Foreign born whites	13,345,545
Negro	9,827,763
All others	412,547
Total population	91,972,267

$\frac{49,488,575}{91,972,267} = .538$ to nearest thousandth

$.538 \text{ of } 360° = 194°$

$\frac{18,897,837}{91,972,267} = .205$ to nearest thousandth

$.205 \text{ of } 360° = 74°$

$\frac{13,345,545}{91,972,267} = .145$ to nearest thousandth

$.145 \text{ of } 360° = 52°$

$\frac{9,827,763}{91,972,267} = .107$ to nearest thousandth

$.107 \text{ of } 360° = 38°$

$\frac{412,547}{91,972,267} = .005$ to nearest thousandth

$.005 \text{ of } 360° = 2°$

CHECK: $\overline{1.000}$ $\overline{360°}$

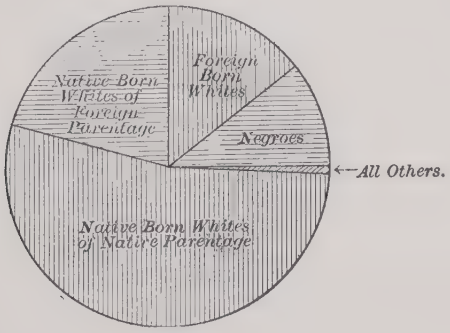

FIG. 49

EXERCISES

1. Represent by a circular graph the tax rate of your city with its divisions for different purposes.

2. In New York City, Census 1910, 19.3% of the total population were native white of native parentage; 38.2% were native white of foreign parentage; 40.4% were foreign born white; and 2.1% were colored. Represent these data by a circular graph.

3. Represent by a circular graph the following data concerning the average number of wage earners (1909) in the United States, grouped according to the different kinds of employment:

Textile industry	1,437,258
Iron and steel industry	1,025,044
Lumber industry	907,514
Paper and printing industry	415,990
Clay, glass, and stone industry	342,827
Leather industry	309,575
Preparation of food, farming, etc.	411,575
All others	1,765,263
Total number	

CHAPTER VI

SUMMARY — MISCELLANEOUS EXERCISES

1. Practice giving at sight the sum of each of the following pairs of numbers.

[SUGGESTION. Add the tens' column first.]

12	22	33	44	55	66	77	88
11	11	11	11	11	11	11	11
23	33	44	55	66	77	21	22
22	22	22	22	22	22	13	13
14	24	23	25	26	24	34	54
13	13	13	13	13	14	14	14
25	45	35	65	32	47	71	81
14	14	14	14	23	32	28	26
45	72	35	64	57	96	63	66
32	17	33	35	52	12	42	43
46	67	93	72	46	97	72	84
38	35	21	18	23	15	41	62
71	89	78	93	77	89	98	99
29	35	26	41	44	27	72	89

Also practice giving at sight the difference between each pair of numbers.

2. Practice giving at sight the sum of each of the following pairs of fractions always expressing the answer in lowest terms.

$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{7}{8}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$\frac{1}{4}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{7}{16}$
$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{3}{4}$
$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{1}{16}$	$\frac{3}{16}$
$\frac{1}{3}$	$\frac{2}{3}$	$\frac{5}{6}$	$\frac{5}{6}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{8}{9}$
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{5}{9}$	$\frac{2}{3}$
$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$
$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{7}$	$\frac{1}{5}$	$\frac{1}{8}$	$\frac{1}{5}$	$\frac{1}{6}$
$\frac{1}{2}$	$\frac{5}{6}$	$\frac{1}{6}$	$\frac{5}{6}$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{5}{6}$	$\frac{7}{8}$
$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{5}{6}$

Also practice giving the difference between each pair of fractions.

3. Practice giving the fractional parts of the numbers on the rim of the circle in Fig. 50.

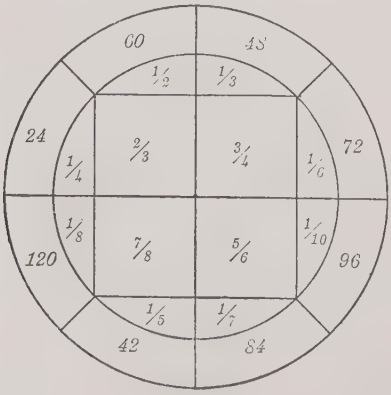

FIG. 50

4. Practice giving per cents of the numbers on the rim of the circle in Fig. 51.

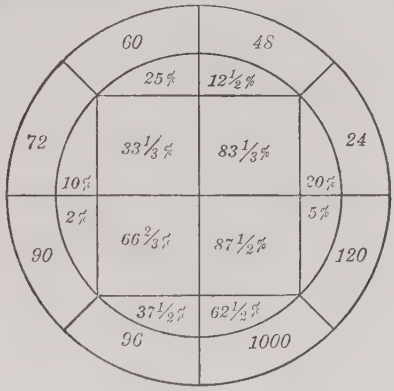

FIG. 51

5. Add as decimal fractions each of the following pairs of fractions.

$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	.6	.7	.8
.1	.2	.3	.4	.5	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
.9	$\frac{1}{5}$	$\frac{1}{5}$	.3	.4	.5	.6	.7
$\frac{1}{2}$	.1	.2	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$
$\frac{2}{5}$	$\frac{2}{5}$	.5	$\frac{2}{5}$	.7	$\frac{2}{5}$	.9	.8
.1	.3	$\frac{2}{5}$	.2	$\frac{2}{5}$	.4	$\frac{2}{5}$	$\frac{2}{5}$
$\frac{3}{5}$	$\frac{3}{5}$	.7	$\frac{3}{5}$	.9	.8	$\frac{3}{5}$	$\frac{3}{5}$
.1	.3	$\frac{3}{5}$	.5	$\frac{3}{5}$	$\frac{3}{5}$	.2	.6
$\frac{4}{5}$	$\frac{4}{5}$	$\frac{4}{5}$	.9	$\frac{4}{5}$	.8	$\frac{4}{5}$	$\frac{4}{5}$
.1	.2	.6	$\frac{4}{5}$	.5	$\frac{4}{5}$	.7	.4

Find in the same way the difference between each pair of fractions.

6. Add as common fractions each of the following pairs of fractions.

$\frac{1}{3}$	.75	$\frac{2}{3}$	.75	.25	$\frac{5}{6}$	$\frac{5}{6}$	.25
.25	$\frac{1}{3}$	.25	$\frac{2}{3}$	$\frac{1}{6}$	.25	.75	$\frac{1}{7}$
.25	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{7}{8}$	.75	.75	.75	$\frac{7}{8}$
$\frac{1}{8}$	.25	.25	.25	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{5}{8}$	.75

Find also the difference between each pair of fractions.

7. In the following table: (*a*) Compute the average price, in cents, of a bushel of corn in each state for the years 1914, 1915, and 1916. (Divide the sum of the *three* prices in each line by 3; work out each quotient to *tenths;* then write the answer to the nearest cent. Thus 72.8 cents would be 73 cents; 72.4 cents would be 72 cents.) (*b*) In a similar way compute the average yield in bushels per acre, correct to *tenths*, for the years 1914, 1915, and 1916. (As these numbers are given correct to *tenths*, work out each quotient to *hundredths*, and then write the answer to the nearest *tenth*. Thus 18.47 bu. would be 18.5 bu.; 18.43 bu. would be 18.4 bu.)

UNITED STATES CROP REPORT, DECEMBER, 1916

THE SIX LEADING CORN PRODUCING STATES	PRICE PER BU. DECEMBER 1			AVERAGE	YIELD PER ACRE			AVERAGE
	1914	1915	1916		1914	1915	1916	
	cts.	cts.	cts.	cts.	bu.	bu.	bu.	bu.
Iowa . . .	55	51	80		38.0	30.0	36.5	
Illinois . . .	61	54	84		29.0	36.0	29.5	
Nebraska . .	53	47	78		24.5	30.0	26.0	
Indiana . . .	58	51	84		33.0	38.0	34.0	
Missouri . . .	68	57	90		22.0	29.5	19.5	
Texas . . .	74	58	104		19.5	23.5	19.0	

8. In the following table:

(*a*) Find the sum of each column.

(*b*) Find the average number of acres planted.

(*c*) Find the average yield per acre.

(*d*) Find the average price per bushel.

CROP STATISTICS, DECEMBER 1, 1916

Potatoes

STATE	ACREAGE	YIELD PER ACRE	PRICE PER BUSHEL
	acres	bushels	cents
Maine	125,000	204	142
New Hampshire	15,000	120	166
Vermont	23,000	112	139
Massachusetts	25,000	91	175
Rhode Island	5,000	74	185
Connecticut	22,000	95	175
New York	320,000	70	158
New Jersey	85,000	122	155
Pennsylvania	272,000	70	148
Delaware	10,000	90	125
Maryland	43,000	95	133
Virginia	125,000	130	137
West Virginia	48,000	117	158
North Carolina	34,000	90	140
South Carolina	10,000	80	175
Georgia	15,000	60	175
Florida	15,000	74	200
Ohio	140,000	45	182
Indiana	74,000	44	177
Illinois	125,000	58	179
Total	?	?	?
Average	?	?	?

The numbers and prices for problems 9–18 inclusive, that follow, will be found in the above table.

9. If the average yield of an acre in Maine was 204 bushels of potatoes and the price per bushel, Dec. 1, 1916, was $1.42, find the value of an acre of potatoes in the state of Maine.

10. Find the value of an acre of potatoes in New Hampshire. (Consult table.)

11. Find the value of an acre of potatoes in Vermont.

12. Find the value of an acre of potatoes in Massachusetts.

13. Find the value of an acre of potatoes in Rhode Island.

14. Find the value of an acre of potatoes in New Jersey.

15. What was the excess in value of an acre of potatoes in Maine over that of an acre in New Hampshire? (Compare answers of Exs. 9 and 10.)

16. What is the excess in value of an acre of potatoes in New Hampshire over that of an acre in Vermont? In New York over that in Pennsylvania?

17. Find the number of bushels of potatoes produced in Rhode Island in 1916. In Connecticut. In Delaware.

18. Find the value of the crop of potatoes in Rhode Island in 1916. In Delaware.

19. In 1915 New York produced 22,010,000 bushels of potatoes. The average yield per acre was 62 bushels. How many acres were planted?

20. In 1915 Maine produced 25,418,000 bushels of potatoes. The average yield per acre was 179 bushels. How many acres were planted?

21. In the following table find the total of each column and the average.

Monthly Prices of Wheat

The following table gives in the United States the estimated average price, cents per bushel, to producers of wheat, monthly for 9 years.

	1916	1915	1914	1913	1912	1911	1910	1909	1908
Jan. 1 . . .	102.8	107.8	81.0	76.2	88.0	88.6	103.4	93.5	88.7
Feb. 1 . . .	113.9	129.9	81.6	79.9	90.4	89.8	105.0	95.2	89.0
Mar. 1 . . .	102.9	133.6	83.1	80.6	90.7	85.4	105.1	103.9	89.2
Apr. 1 . . .	98.6	131.7	84.2	79.1	92.5	83.8	104.5	107.0	89.8
May 1 . . .	102.5	139.6	83.9	80.9	99.7	84.6	99.9	115.9	89.3
June 1 . . .	100.0	131.5	84.4	82.7	102.8	86.3	97.6	123.5	92.3
July 1 . . .	93.0	102.8	76.9	81.4	99.0	84.3	95.3	120.8	89.5
Aug. 1 . . .	107.1	106.5	76.5	77.1	89.7	82.7	98.9	107.1	90.4
Sept. 1 . . .	131.2	95.0	93.3	77.1	85.8	84.8	95.8	95.2	88.7
Oct. 1 . . .	136.3	90.9	93.5	77.9	83.4	88.4	93.7	94.6	90.4
Nov. 1 . . .	158.4	93.1	97.2	77.0	83.8	91.5	90.5	99.9	91.5
Dec. 1 . . .	160.3	91.9	98.6	79.9	76.0	87.4	88.3	98.6	92.8
Total . . .									
Average . .									

22. Find the cost of 825 bushels of wheat at the price of Jan. 1, 1916.

23. Find the cost of 825 bushels of wheat at the price of Dec. 1, 1916.

24. Find the excess of the cost in Ex. 23 over that in Ex. 22.

25. A farmer had 384 bushels of wheat to sell. He sold $\frac{2}{3}$ of it at \$1.36 per bushel, the remainder at \$1.58. What was the total amount received for his crop of wheat?

26. A farmer had 360 bushels of corn to sell. He sold 25% of it at 78 cents per bushel, $33\frac{1}{3}\%$ of what was left at 80 cents per bushel, the remainder at 75 cents per bushel. How much did he receive for his corn?

27. An automobile covers 242 miles in 12 hours. Taking out $1\frac{1}{4}$ hours, the time consumed in getting dinner and making repairs, what was the average number of miles actually driven per hour?

28. A tank has a capacity of 7 cu. ft. How many gallons will it hold? (231 cu. in. = 1 gal.)

29. A car of 220 bushels of potatoes was loaded in Northern Maine. The potatoes cost $1.20 per bushel on the car. On the way to New York City 10 per cent of them were frost-bitten. If the potatoes that were saved were sold on the car in New York City for $1.50 a bushel, what was the net profit on this transaction?

30. 100 yards are what per cent of 440 yards?

31. 220 yards are what per cent of 100 yards?

32. A laborer gets d dollars per week. Express the amount he will receive for 3 weeks; $2\frac{1}{2}$ weeks; $\frac{1}{2}$ week.

33. At the wage in Ex. 32 how will you express what the laborer gets per day? (Call a week 6 days.) For 2 days? For 3 days? Solve Exs. 34–38 by the equation method. (See Chapter II.)

34. A's salary in 1916 was $3762. This was $\frac{4}{5}$ of his salary in 1915. Find his salary for 1915.

35. The registration in school A was 564 pupils in 1916. This was $\frac{3}{4}$ of the registration in 1915. What was its registration in 1915?

36. When coal retails for $9.50 per ton (2000 lb.), how much should I pay for 2500 lb.? For 3500 lb.? For 5000 lb.?

37. Express the following as a ratio per hundred.

(*a*) $\frac{3}{4}$; $\frac{12}{16}$; $\frac{8}{32}$; $\frac{2.4}{4.8}$; $\frac{3.2}{6.4}$; $\frac{1.2}{3.6}$.

(*b*) $\frac{1}{3}$; $\frac{2}{3}$; $\frac{1}{6}$; $\frac{5}{6}$; $\frac{6}{18}$; $\frac{.3}{.9}$.

38. At \$38.40 per 1000 ft. what will be the cost of each of the following amounts of lumber: 250 ft.? 500 ft.? 750 ft.? 375 ft.? 625 ft.? 875 ft.? Find the total cost.

39. If a 10% discount was given on the amount of the bill in the previous exercise, find the net amount of the bill.

40. At \$52.00 per 1000 ft. find the cost of each of the following amounts of lumber: 210 ft.; 125 ft.; 40 ft.; 25 ft.; 22 ft. Find the total cost.

41. The following are advertised to be sold at a discount of 20% on the marked price. Determine the discount and the net price of each article.

Article	Marked Price	Discount	Net Price
a. Overcoat .	\$12.50	?	?
b. Suit . . .	\$22.50	?	?
c. Suit . . .	\$14.50	?	?
d. Overcoat .	\$18.50	?	?
e. Overcoat .	\$22.50	?	?
f. Suit . . .	\$27.50	?	?
g. Suit . . .	\$32.50	?	?
h. Overcoat .	\$42.00	?	?
i. Overcoat .	\$47.50	?	?
j. Overcoat .	\$52.50	?	?

42. The price of an automobile is raised from \$960 to \$1070. What is the per cent of increase?

43. The price of an automobile is raised from \$1090 to \$1250. What is the per cent of increase?

44. The price of an automobile is raised from \$1250 to \$1450. What is the per cent of increase?

45. An agent's commission for selling automobiles is 15% of the selling price. Determine his commission on each of the following:

(*a*) An automobile selling for \$1070.

(*b*) An automobile selling for \$1250.

(*c*) An automobile selling for \$1450.

46. Determine the total amount earned by an agent who sold during July, 5 automobiles of grade (*a*), 3 of grade (*b*), and 2 of grade (*c*), in Ex. 45.

47. Compute the simple interest on \$1450 for 9 mo. at $4\frac{1}{2}$ per cent.

48. Compute the simple interest on \$2560 for 11 mo. 15 da. at 5 per cent.

49. Find the amount due on \$1250 for 8 mo. 25 da. at 4 per cent simple interest.

50. Find the amount due on \$2275 for 7 mo. 12 da. at $4\frac{1}{2}$ per cent simple interest.

51. Find the amount due on \$3250 for 10 mo. 25 da. at $6\frac{1}{2}$ per cent simple interest.

52. In a certain store it is customary to allow each clerk, in addition to his regular salary, a commission of 10 per cent on the excess of the gross amount of his sales for the week over \$1000. Determine the excess over \$1000 and each clerk's commission in the following list:

Clerk's Number	Amount of Sales	Excess over \$1000	Commission
No. 11	\$1025.75	\$25.75	\$2.58
No. 15	1115.75	?	?
No. 21	1505.50	?	?
No. 22	1800.00	?	?
No. 23	1985.60	?	?
Total . .	?	?	? (Check)

53. A concrete walk 72 feet long and $4\frac{1}{2}$ feet wide is laid at an expense of 24 cents per square foot. A discount of 5% is allowed if the work is paid for when completed. How much cash will be needed, if payment is to be made when the work is completed?

54. The city decides to lay a concrete walk on *both* sides of a street and requires all property owners on this street to pay $\frac{2}{3}$ of the expense of laying the walk. The street is $\frac{3}{4}$ mile long and the walk is to be 4 feet wide. The cost of the walk is 23 cents per square foot. Determine the entire cost of the walk, and the amount that the property owners must pay.

55. A owns a lot which has a frontage of 125 feet on this street. How much will he have to pay?

56. B owns a lot which has a frontage of 75 feet on the same street. How much will he have to pay?

57. A rug 12 ft. 2 in. long and 9 ft. 10 in. wide is placed in a room 21 ft. long and 15 ft. wide.

(*a*) Find the number of square feet of floor space covered by the rug (to the nearest square foot).

(*b*) Find the number of square feet of floor space in the room (to the nearest square foot).

(*c*) Find the per cent of floor space covered by the rug.

(*d*) Find the per cent of floor space not covered by the rug.

58. There are six windows in the room whose dimensions are given in Ex. 57; each window is 53 in. high and 27 in. wide.

(*a*) Find to the nearest thousandth the ratio of the total window area to the floor area.

(*b*) Find the per cent the total window area is of the floor area.

59. In this room there are two open entrances; one is 6 ft. 8 in. high and 6 ft. wide, the other is 6 ft. 8 in. high and 4 ft. wide.

(*a*) Find to the nearest thousandth the ratio of the total door space to the floor area.

(*b*) Find the ratio of the total door space to the window area.

60. Cedar posts, costing 22 cents each, are to be set around the two sides and one end of a rectangular lot of land. The lot is 330 feet long and 135 feet wide and the posts are to be set 15 feet apart. Find the number of posts needed and the cost. (Make a diagram.)

61. Five wires are to be strung on the posts in Ex. 60. If the cost of the wire is $2\frac{1}{2}$ cents per linear foot, find the cost of the wire needed, adding 20 feet for waste.

62. How long must a strip of land 220 yd. wide be to contain $1\frac{1}{2}$ acres? [Use the formula for the area of the rectangle.]

63. A lot of land 230 feet long by 70 feet wide is bought at 17 cents per square foot. Find the cost.

64. A house lot in the shape of a trapezoid is bought at 18 cents per square foot. The width of the lot is 68 feet; one of the parallel sides is 110 ft. the other 122 ft. Find the cost of the lot.

For Ex. 65–67, see Table, Ex. 8.

65. By line-graphs compare the yield in bushels per acre. Let 1 inch represent 100 bushels.

66. By line-graphs compare the price in cents per bushel. Let 1 inch represent 100 cents.

67. By line-graphs compare the acreage of potatoes in the twenty states. Let 1 inch represent 50,000 acres.

68. The table gives the distances from New York via water to each of the following places. Find the distance saved and the per cent of distance saved to the nearest per cent.

	Former Route	Panama Canal	Distance Saved	Per Cent Saved
	miles	miles	miles	
San Francisco .	13,135	5,262	?	?
Hawaii	12,800	7,000	?	?
Manila	17,800	12,000	?	?

69. The following budget gives the proper distribution, based upon thousands of careful tests, of a monthly income of $100 for each of the following items:

Food — Groceries, meats, provisions, milk, etc.	$31.00
Clothing and house furnishings generally . .	15.00
Rent, light, and heat	24.00
Car fare, life insurance, doctors, medicines . .	12.00
Amusements, recreation, vacation, etc. . . .	8.00
Savings and contingent funds	10.00
	$100.00

Represent this distribution by a circular graph. (See page 122.)

70. Copy the items from the budget of the family income, Ex. 69, and then determine how much should be set aside for each purpose:

(*a*) If the yearly income is $ 600.

(*b*) If the yearly income is $ 800.

(*c*) If the yearly income is $1200.

(*d*) If the yearly income is $1500.

71. If the family income is $1350 per year, how much should be set aside for rent, light, and heat? If the bills

for light and heat average $5 per month, what is the limit of monthly rent that this family can pay?

72. How much can the above family afford to spend for amusement, etc.? How much should they save?

73. The popular vote for President of the United States in November, 1916, was: Wilson, 9,068,894; Hughes, 8,568,051. Find the total popular vote. What per cent of the total vote did Wilson receive? What per cent did Hughes receive?

74. The following articles are advertised to be sold at a discount of 15% from the marked price. Determine the net price in each instance:

(*a*) Coat, $18.00.
(*b*) Suit, $25.00.
(*c*) Overcoat, $30.00.
(*d*) Overcoat, $35.00.
(*e*) Overcoat, $40.00.

75. An army aviator traveled 190 miles in $3\frac{1}{4}$ hours. What was his average rate, in miles per hour?

76. In October, 1904, a New York Central train went 3.51 miles, from Croton to Ossining, in 2 minutes. What was the average rate, in miles per hour?

77. In 1897 a railroad train went from Chicago to Denver, a distance of 1,025 miles, in 18 hours 52 minutes. What was the average rate, in miles per hour?

78. In a certain class 8 of every 10 pupils pass. In this class there are 35 pupils. How many pass?

79. In a certain school 3 of every 7 pupils have defective vision. If there are 840 pupils in this school, how many have defective vision?

80. A hockey team won 3 of every 5 games it played. If this team won 12 games, how many games did it play?

81. In the following table determine the ratio of the total window area to the floor area in each room and express it decimally to the nearest hundredth.

Room	Dimensions of Window	Number of Windows	Dimensions of Room	Ratio
No. 1	6 ft. 3 in. by 3 ft.	6	25 ft. by 18 ft.	?
No. 2	6 ft. 3 in. by 3 ft.	7	27.6 ft. by 20 ft.	?
No. 3	6 ft. 3 in. by 3 ft.	10	32.5 ft. by 30.4 ft.	?
No. 4	6 ft. 3 in. by 3 ft.	8	30.6 ft. by 28.5 ft.	?

APPENDIX

REFERENCE-TABLES

1. Linear Measure

12 inches (in.) = 1 foot (ft.).
3 feet = 1 yard (yd.).
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet = 1 rod (rd.).
320 rods, or 5280 feet = 1 mile (mi.).
6 feet = 1 fathom.
1.151 miles = 1 knot.

2. Square Measure

144 square inches (sq. in.) = 1 square foot (sq. ft.).
9 square feet = 1 square yard (sq. yd.).
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.).
160 square rods = 1 acre (A.).
640 acres = 1 square mile.
36 square miles = 1 township.

3. Cubic Measure

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.).
27 cubic feet = 1 cubic yard (cu. yd.).
16 cubic feet = 1 cord foot (cd. ft.).
128 cubic feet = 1 cord (cd.).

4. Dry Measure

2 pints (pt.) = 1 quart (qt.).
8 quarts = 1 peck (pk.).
4 pecks = 1 bushel (bu.).

5. Liquid Measure

4 gills (gi.) = 1 pint.
2 pints = 1 quart.
4 quarts = 1 gallon (gal.).
63 gallons = 1 hogshead.

6. Avoirdupois Weight

16 ounces (oz.) = 1 pound (lb.).
2000 pounds = 1 ton (T.).
32 pounds = 1 bushel of oats.
48 pounds = 1 bushel of barley.
56 pounds = 1 bushel of corn or rye.
60 pounds = 1 bushel of wheat or potatoes.
196 pounds = 1 barrel (bbl.) of flour.
2240 pounds = 1 long ton of coal.

7. United States Money

10 mills (m.) = 1 cent (ct. or ¢).
10 cents = 1 dime (di.).
10 dimes = 1 dollar ($).
100 cents = 1 dollar.
10 dollars = 1 eagle.

8. Time

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.).
24 hours = 1 day (da.).
7 days = 1 week (wk.).
365 days = 1 common year (yr.).
366 days = 1 leap year.
12 months = 1 year.
360 days = 1 commercial year.
100 years = 1 century.

9. Counting

12 units = 1 dozen (doz.).
12 dozen = 1 gross (gro.).
24 sheets of paper = 1 quire.
500 sheets = 1 ream.

10. Circular Measure

60 seconds (″) = 1 minute (′).
60 minutes = 1 degree (°).
360 degrees = 1 circumference.
69⅛ miles = 1 degree of latitude.

Equivalents

1 bushel = $\frac{5}{4}$ cu. ft., or 2150.42 cu. in.
1 gallon = 231 cu. in.
1 cu. ft. = $7\frac{1}{2}$ gal. (approx.).
1 ton of hay = 500 cu. ft. (approx.).
1 ton of coal = 35 cu. ft. (approx.).

FORMULAS

I. Notation Used in Formulas

A = area.
b, b_1, b_2 = bases.
h = height or altitude.
s = side.
c = circumference.
d = diameter.
r = radius.

II. Formulas

1. Area of a rectangle, $A = bh$, p. 92.
2. Area of a square, $A = s^2$, p. 100.
3. Area of a parallelogram, $A = bh$, p. 104.
4. Area of a triangle, $A = \frac{1}{2}bh$, p. 106.
5. Area of a trapezoid, $A = \frac{1}{2}(b_1 + b_2)h$, p. 112.
6. Circumference of a circle, $c = \pi d$ or $2\pi r$, p. 118.
7. Area of a circle, $A = \pi r^2$, p. 119.

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